A Simulation for Studying the Collisional Dusty Plasma Sheath Phenomenon

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Abstract

A mathematical and computer simulation have been achieved for the collisional processes in dusty plasma sheath, including the effect of power law dependence for cross section on particles energy. Based on this model exact numerical solution of sheath governing equations using Runge – Kotta' routines are obtained for special case of constant dust mobility. The effect of collisinal parameters on electric potential sheath width and average impact energy at wall for both ion – electron and dusty plasmas have been studied by numerically solving the characteristic equations. It is found that different charge numbers on the dust grains, the ratio of ion to electron densities, and the electric potential (and hence the electric field) have a strong effect on the dusty plasma sheath. A comparison between exact numerical solution and derivate analytical approximate expressions for sheath width and impact energy are made.

Introduction

A plasma sheath is characterized by the localized electric field that separates a plasma particles from a material boundary. The negative boundary potential is due to the higher thermal velocities of electrons that hit the wall and lose faster than ions leaving the plasma at positive potential with respect to the wall. This potential cannot be distributed normally over the entire plasma, since the Debye shielding will confine the potential variation to a layer of the order of several Debye lengths in thickness. This layer, which must exist on all cold boundary walls (with which the plasma is in contact), is called an electrostatic sheath [1].

We adopt the previously model given by Ref. [2], to study the effect of collisionality on sheath edge potential, sheath width, and impact energy. Thus, we consider unmagnetized steady state dusty plasma which consists of thermal electrons, thermal ions, micron size of negatively charged dust grains and neutral particles. Also we assume that the sheath parameters are time independent and as such we ignore any instabilities or waves in the sheath. The electrons and ions being much lighter than the dust particles are assumed to be in thermal equilibrium with dusts as a cold fluid, and neutrals are taken to be immobile. Although the dusts are massive with respect to the ions and electrons, due to their inertia and positive ion – dust interactions, the dusts will possess a drift velocity. However, the negative potential will repel the negatively charged dusts which are moving towards the wall. The dust density will become more at the sheath plasma boundary; because, inertial dusts moving from the bulk plasma towards the wall and the dusts repelled by the wall potential will be accumulated in that region. As a result, dust will collide with the immobile neutrals at the sheath region. This dust – neutral collisions change the sheath behavior of the plasma widely.

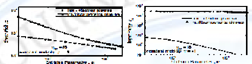
In the two – component plasmas model several authors have considered the effect of ions collisionality on the sheath. Godyak and Sternberg [3] presented a fluid model where the ions experience a collisional drag. Sheridan and Goree [4] studied the amount of collisionality needed to cause the transition from the collisionless to the collisionally dominated regime. The latter have derived a general formula for the plasma sheath which is used here for comparison with dusty plasma sheath formula.

It is expected that the presence of dust charged particles will lead to a very different behavior of the sheath as compared to that of ordinary plasma. For instant, in material plasma processing techniques where negatively charged dust particles are usually found to be present, the collisions of the dust charged particles with the neutrals are expected to exhibit new features. The propose of this work is to study the effect of collisionality on sheath edge potential, sheath width, and impact energy in dusty plasma and to compare this effect to that of ion – electron plasma. Moreover, approximate formulae for the parameters that affect collisional sheath are sometime used. Hence comparisons between these formulae and exact numerical solutions are to be done to examine where they are dropping across.

The organization of the paper is as follows. In Sec. II the theory presents fluid equations with a power law dependence on the dust speed of the collisional dusty plasmas sheath. The results from the numerical solutions are discussed in Sec. III and Sec. IV is the presented approximated analytical solutions. Sec. V is the conclusion.

Theory

Since electrons and ions are assumed to be in thermal equilibrium, accordingly, their densities are obey to the Boltzmann relation


(1)

$$n_i = n_{i0} \exp\left(\frac{-e\phi}{\kappa T_i}\right) \tag{2}$$

where e is the electronic charge, κ is Boltzmann's constant, ϕ is the potential in the sheath, n_{e0}, n_{i0} are respectively the densities of the electrons and ions at the sheath edge. T_e , T_i are the electron and ion temperature, respectively. In this paper SI system units used for entire expressions and constants.

Because the sheath is source free, the dust density obeys the following equation of continuity

$$\nabla \cdot (n_d \mathbf{v}_d) = 0 \tag{3}$$

where n_d, $\mathbf{v}_d$ and Z_d are respectively the density, velocity and charge number of dust particle.

The equation of motion,

$$m_d(\mathbf{v}_d \cdot \nabla)\mathbf{v}_d = Z_d \nabla\phi - \mathbf{F}_c \tag{4}$$

The collisional a drag force term $\mathbf{F}_c$ used in eq. (4), is given by [5]

$$\mathbf{F}_c = m_d(n_n \sigma v_d)\mathbf{v}_d \tag{5}$$

where n_n is the neutral gas density and σ is the momentum transfer cross section for collisions between dusts and neutrals. Elastic and charge-exchange collisions contribute to this cross section, which depends on the relative dust velocity $\mathbf{v}_d$.

Poisson' equation

$$\varepsilon_0 \nabla^2 \phi = -e(n_i - n_e - Z_d n_d) \tag{6}$$

where ε_0 is the permittivity constant. n_i, n_e and n_d are respectively the ion, electron and dust densities. To complete the set of equations we must specify the dependence of the cross-section on dust energy. We assume that it has a power law dependence on the dust speed of the form [6]

$$\sigma(v_d) = \sigma_s \left(\frac{v_d}{C_d}\right)^{\gamma} \tag{7}$$

where $C_d = (\kappa T_e/m_d)^{1/2}$ is the dust acoustic speed and γ is a dimensionless parameter ranging from 0 to -1. The power law holds $\gamma = 0$ for constant dust mean free path l_{mfp}, in which $l_{mfp} \gg \lambda_D$, where λ_D is Debye length, and $\gamma = -1$ for mobility limited case, $l_{mfp} \ll \lambda_D$ [7]. As mentioned above we concern with the last case.

Combining Equations (1 – 7), we find two coupled, differential equations describing the planar plasma sheath:

$$v_d \frac{dv_d}{dx} = \frac{Z_d e}{m_d}\frac{d\phi}{dx} - n_n \sigma_s \frac{v_d^{\gamma+2}}{C_d^{\gamma}} \tag{8}$$

$$\nabla^2 \phi = -\frac{n_{e0}}{\varepsilon_0}\left[\delta e^{-\theta e\phi/\kappa T_i} - e^{e\phi/\kappa T_e} - (\delta - 1)\frac{v_{d0}}{v_d}\right] \tag{9}$$

In which the dusty plasma quasineutrality condition for negatively charged dust grains $n_{i0} = n_{e0} + Z_{d0} n_{d0}$ holds above and we assume $\theta = \frac{T_e}{T_i}$ and $\delta = \frac{n_{i0}}{n_{e0}}$ then $\frac{Z_{d0} n_{d0}}{n_{e0}} - \delta - 1$.

The governing equations can be made dimensionless by an appropriate choice of variables [8]. The electric potential ϕ is scaled by the electron temperature $\eta \equiv -e\phi/\kappa T_e$, the distance x is scaled by the Debye length $\xi \equiv x/\lambda_D$, and the dust velocity v_d is scaled by the dust acoustic speed, $u \equiv v_d/C_s$. Additionally, the dust kinetic energy is made dimensionless by the electron thermal energy, such that $\mathcal{E} = u^2/2$, so the dimensionless dust impact energy at the wall is $\mathcal{E}_w = u_w^2/2$, where u_w is the dimensionless dust velocity at the wall.

Substituting the dimensionless variables in equations (8) and (9) we obtain

$$u\,u' = -Z_d \eta' - \alpha u^{\gamma+2} \tag{10}$$

$$\eta'' = \delta e^{\eta\theta} - e^{-\eta} + (1-\delta)\frac{u_{d0}}{u} \tag{11}$$

where u' is the first derivative of u and η'' is the second derivative of η with respect to ξ, α is the number of collisions in a Debye length and it is given by $\alpha = \lambda_D n_n \sigma_s$. These two equations, together with appropriate boundary conditions, provide the description of the collisional sheath. To solve these equations boundary conditions must be specified. At the wall ($\xi = d$) the boundary condition is $\eta(d) = \eta_w$. At the sheath-plasma boundary ($\xi = 0$) the boundary conditions are $\eta(0) = 0$, $\eta'(0) = 0$, and $u(0) = u_0$. Note that these conditions are only an approximation to the conditions that actually hold at the sheath plasma interface.

The dimensionless collisional sheath equations for ion – electron plasma are given by [4]

$$uu' = \eta' - \alpha u^{\gamma+2} \tag{12}$$

$$\eta'' = \frac{u_{i0}}{u} - e^{-\eta} \tag{13}$$

It is clear that the variables Z_d, δ and θ are the main differences between the dusty and the ion – electron plasma equations. This is due to presence of dust grain particles in the ion – electron plasma.

NUMERICAL SOLUTIONS

Since there is no closed form analytic solution for governing equations one must either use an approximate analytic solution or exact numerical solution to determine electric potential η in the sheath. The governing equations are solved exactly (i.e., without any approximations) in both cases; two – component ion – electron and dusty plasmas for the electric potential $\eta(\xi)$ and ion velocity $u(\xi)$ by integrating them numerically with a Runge-Kutta' routine [9]. Firstly, the numerical results for the collisional case have been investigated in ion – electron plasma in order to test the numerical code against previous numerical results. The results agree with the Ref. [4].

It is, however, necessary to refer that exact numerical solution of dusty plasma for this model are not worked in References [2, 10]. Instead of that only approximate solutions for sheath width and average impact energy are plotted. Figures 1, 2 and 3 illustrate exact numerical solutions of dusty plasma for sheath thickness and the dust impact energy. In Fig. 1 we compare between ion – electron and dusty plasmas by plotting the sheath thickness d and the dust impact energy ε_w as functions of the collision parameter α for constant wall potential $\eta_w = 10$. We assume $u_0 = 1$, since Bohm criterion is sufficient for both dusty and electron – ion plasmas [4, 11]. In both plasmas the sheath width and impact energy are reduced by collisions characterized by collision parameter α . In collisional dominated region, with higher collision parameter α, the ion motion is collisionally dominated; both d and ε_w decrease and approach power law asymptotes. For small α, collisions are small and both d and ε_w are weak dependent of α. However, the presence of dust grains in the sheath region increase the sheath width and impact energy. The increasing of sheath thickness and the dust impact energy in case of dusty plasma is due to the presence of new parameters characterized by the presence of the dust such as the factor Z_d , δ and its power.

It is known that the electric force may increase, decrease depending upon the grain charge and so the electric field and potential [12]. For dusty plasma the higher of charge number of the grain particles results in the higher sheath thickness d and the higher the dust impact energy ε_w due to the larger collisions. This effect is obvious in Fig. 2. Fig.3 shows the increasing of sheath width and impact energy with increased ratio δ. This is due to increase of ion number density in the bulk of dusty plasma. On the other hand the impact energy exiguously increases with Z, thus the curves coincide on each other.

Approximate solutions

We derive expressions that give the potential profile and the thickness for the collisionally dominated sheath. In the limit of strong dust neutral collisions the collision parameter α is large. The equation of motion (Eq. 12) is simplified by neglecting the convective term on the left hand side. Hence the equation becomes

$$\eta' = -\frac{\alpha}{Z_d} u^{\gamma+2} \tag{14}$$

Neglecting $\exp(-\eta)$ and $\delta \exp(\eta\theta)$ terms in Poisson equation, we obtain

$$\eta'' = (1-\delta)\frac{u_{d0}}{u} \tag{15}$$

Using Eq. (14) in the last form, we arrive at a power law solution for η as follows:

$$\eta = \frac{3+\gamma}{5+2\gamma}\left[\frac{3+\gamma}{2+\gamma}u_0(1-\delta)\right]^{\frac{2-\gamma}{3-\gamma}}\left(\frac{-\alpha}{Z_d}\right)^{\frac{1}{3+\gamma}}\xi^{\frac{5+2\gamma}{3+\gamma}} \tag{16}$$

In ion – electron plasma, the power law solution for η is given by the form [4]

$$\eta = \frac{3+\gamma}{5+2\gamma}\left[\frac{3+\gamma}{2+\gamma}u_0\right]^{\frac{2+\gamma}{3+\gamma}}\alpha^{\frac{1}{3+\gamma}}\ \xi^{\frac{5+2\gamma}{3+\gamma}} \tag{17}$$

Obviously, as in two – component ion – electron plasma the collision parameter α appears explicitly in the leading coefficient, but not in the exponent of ξ. The electric potential η varies not only with ξ and α, but also with the energy dependence of the cross section, characterized by γ. In addition to those, the presence of dust grains into the plasma adds another parameters; namely the ratio δ and dust charge number Z_d.

The sheath thickness is obtained by putting the boundary condition $\eta(d) = \eta_w$ as

$$d = \left(\frac{5+2\gamma}{3+\gamma}\right)^{\frac{3+\gamma}{5+2\gamma}}\left[\frac{3+\gamma}{2+\gamma}u_0(1-\delta)\right]^{-\left(\frac{2+\gamma}{5+2\gamma}\right)}\left(\frac{-\alpha}{Z_d}\right)^{\frac{-1}{5+2\gamma}}\eta_w^{\frac{3+\gamma}{5+2\gamma}} \tag{18}$$

In the same boundary, the sheath thickness of ion – electron plasma is given by [4]

$$d = \left[\frac{(5+2\gamma)^{3+\gamma}(2+\gamma)^{2+\gamma}}{(3+\gamma)^{5+2\gamma}}\frac{\eta_w^{3+\gamma}}{\alpha u_0^{2+\gamma}}\right]^{\frac{1}{5+2\gamma}} \tag{19}$$

In the mobility limited case (i.e. $\gamma = -1$) of dusts, equations (17) and (19) simplify to give:

$$\eta = 3^{-1}2^{3/2}u_0^{1/2}\alpha^{1/2}\,\xi^{3/2}\left(\frac{\delta-1}{Z_d}\right)^{1/2} \tag{20}$$

and

$$d = 3^{2/3}2^{-1}u_0^{-1/2}\alpha^{-1/3}\eta_w^{2/3}\left(\frac{Z_d}{\delta-1}\right)^{1/3} \tag{21}$$

Now we wish to find the dust impact energy, which can be written using Eq. (14) as

$$\mathcal{E}_w = \frac{1}{2}u_w^2 = -\frac{1}{2}\left(\frac{Z_d\eta'}{\alpha}\right)^{\frac{2}{2+\gamma}} \tag{22}$$

Evaluating η'_w using Eq. (15) we find

$$\mathcal{E}_w = \frac{1}{2}\left[\frac{5+2\gamma}{2+\gamma}u_0(\delta-1)\frac{Z_d^2}{\alpha^2}\eta_w\right]^{\frac{2}{5+2\gamma}} \tag{23}$$

For the case of constant mobility

$$\mathcal{E}_w = 2^{-1}\times 3^{2/3}u_0^{2/3}\eta_w^{2/3}\alpha^{-4/3}\,Z_d^{4/3}(\delta-1)^{2/3} \tag{24}$$

In Fig.4 we plot the sheath width and average impact energy $\mathcal{E}_w$ as functions of the collision parameter α for various values of electric potential η_w of dusty plasma. The sheath width increases with higher wall potential. Clearly the presence of dust grain influences the ion flow velocities. The difference between ion – electron and dusty plasmas sheath width equations is the factor $[Z_d/(\delta \quad 1)]^{1/3}$. This factor is responsible for increase of sheath width in case of dusty plasma with respect to the ordinary plasma. Likewise, the average impact energy is higher in case of dusty plasma with respect to the ordinary plasma due to the factor $Z_d^{4/3}(\delta-1)^{2/3}$. But in both cases the average impact energy decreases asymptotically. The Figure shows that both the sheath width and average impact energy increases with higher wall potential.

To see how exact numerical solution approaches or departs the approximate solution. Fig. 5 illustrates exact numerical and approximate solutions. Both solutions of sheath width decrease exponentially. However, in small values of collisional parameter the sheath width of exact solution exceeds that of approximate one. But at large values of collisional parameter the exact solution of sheath width is smaller. For the average impact energy the behavior of both solutions with collisional parameter is inverted. The impact energy of exact solution with collisional parameter is slightly decreased, while its behavior in approximate solution is decreasing exponentially in explicit wide range. This is the reason of separation the figure into two diagrams. For the sheath width the exact numerical and approximate solutions of sheath width are approached at nearly $\alpha = 3$ for fixed value of charge number, sheath potential, the ratio δ and u_0 as described in the Fig 5. While for the average impact energy the two solutions are approached at nearly $\alpha = 8$.

Conclusions

A fluid model with a power law dependence on the dust speed is used to study collisional dusty plasma sheath. Comparison of dusty and ion – electron plasmas for the case of collision dominated region is made. Due to dust – neutral collisions features, the sheath width and impact energy in dusty plasma are reduced, compared to that ion – electron plasma. However, in both plasmas, the sheath width and impact energy decrease asymptotically with increase of collision parameter. They increase when the electric potential increases. For given electric potential the sheath width and impact energy increase with charge number on the dust. However, for given charge number and electric potential, sheath width is smaller when the ratio of ion to electron is lower, while the impact energy exiguously decreases. Approximate solutions of this model appropriate for the collisionality – dominated sheath are derived. Comparison of exact numerical and approximate solutions is made.

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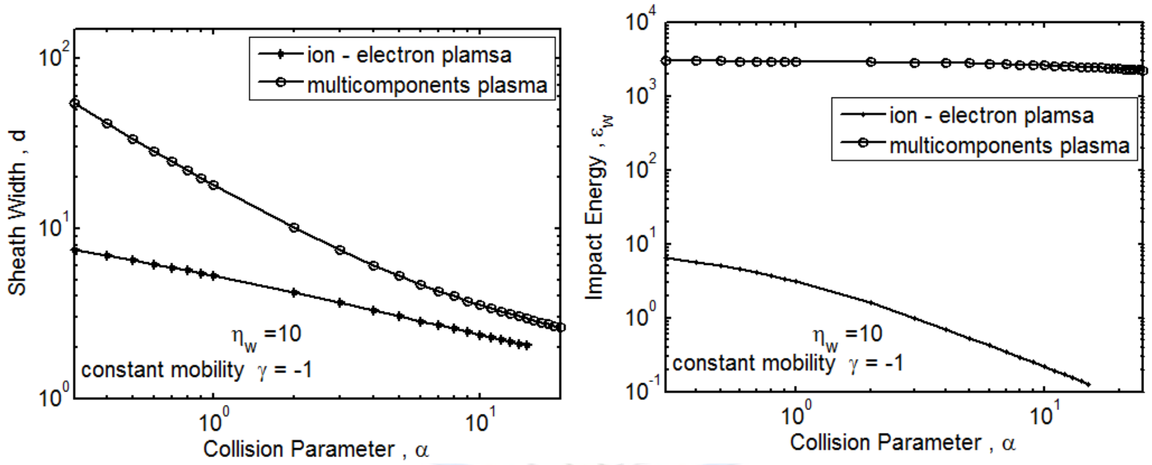

Fig. 1: Exact numerical solutions of the governing equations using Runge – Kotta' routine for the dimensionless sheath thickness d , and average impact energy ε_w as a function of the collision parameter α and constant wall potential η_w at constant mobility in the cases of ion – electron and dusty plasmas. We have assumed $u_0 = 1$ and $Z = 300$ for dusty plasma. The figures show that introducing dusts in two – components plasma raise the sheath width and impact energy.

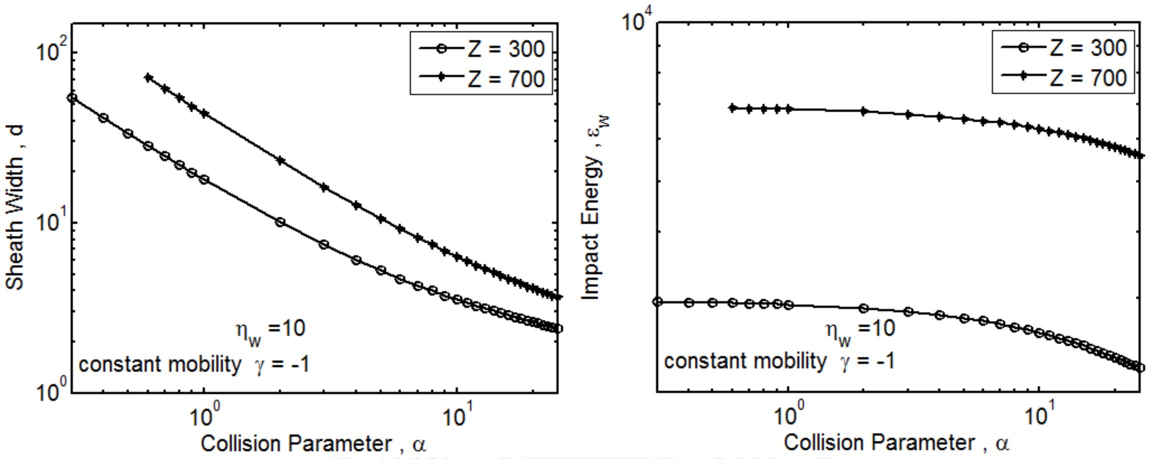

Fig. 2: Exact numerical solutions of the governing equations using Runge – Kotta' routine for the dimensionless sheath thickness d , and average impact energy ε_w as a function of the collision parameter α and constant wall potential η_w at constant mobility in the case of dusty plasmas. We have assumed $u_0 = 1$. The figures show how charge number on the dust particles varies the sheath width and impact energy.

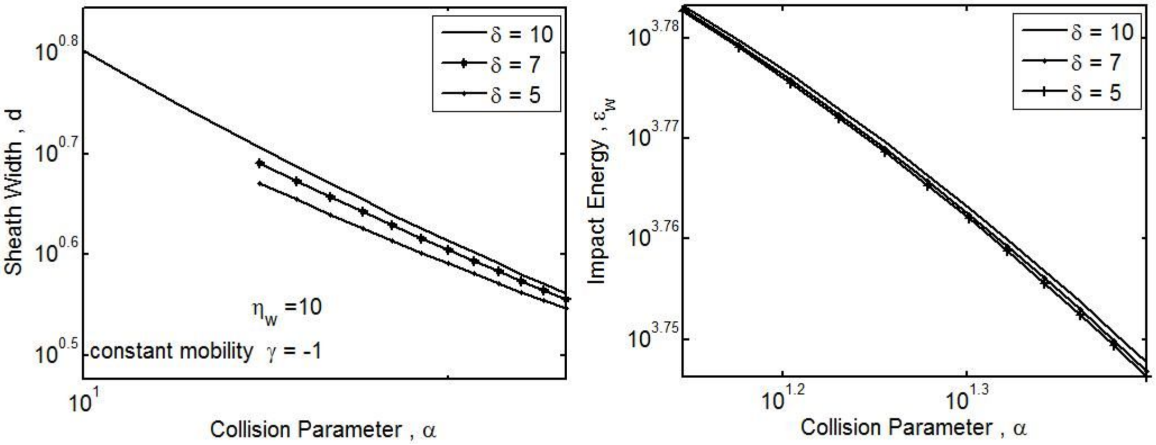

Fig. 3: Exact numerical solutions of the governing equations using Runge – Kotta' routine for the dimensionless sheath thickness d , and average impact energy ε_w as a function of the collision parameter α and constant wall potential η_w at constant mobility in the case of dusty plasmas. We have assumed $u_0 = 1$ and use $Z = 700$ on dust particles.

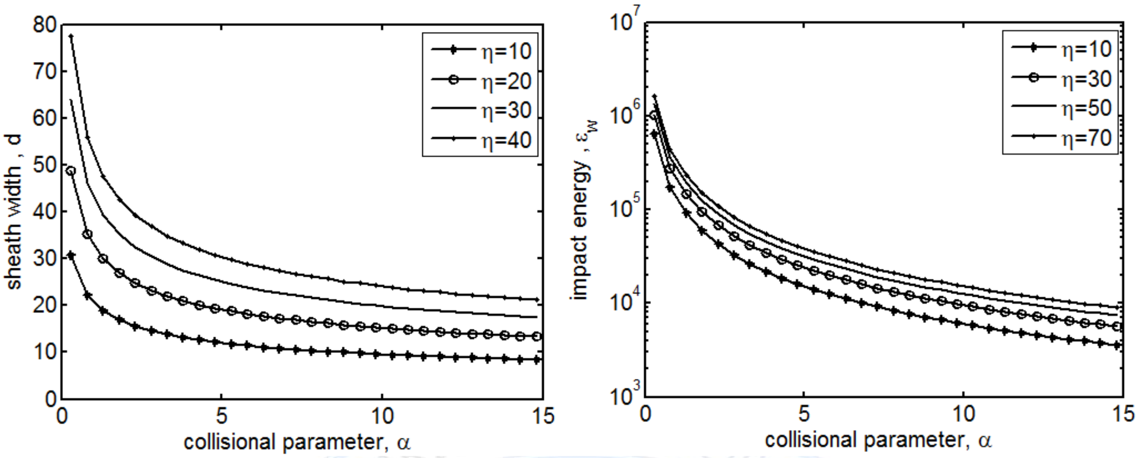

Fig. 4: Approximate solutions of the dimensionless sheath thickness d and average impact energy as a function of the collision parameter α for wall potentials for various values of wall potentials in dusty plasma at constant mobility. In the calculations we use charge number $Z = 700$. The sheath width and impact energy increases with increasing electric potential.

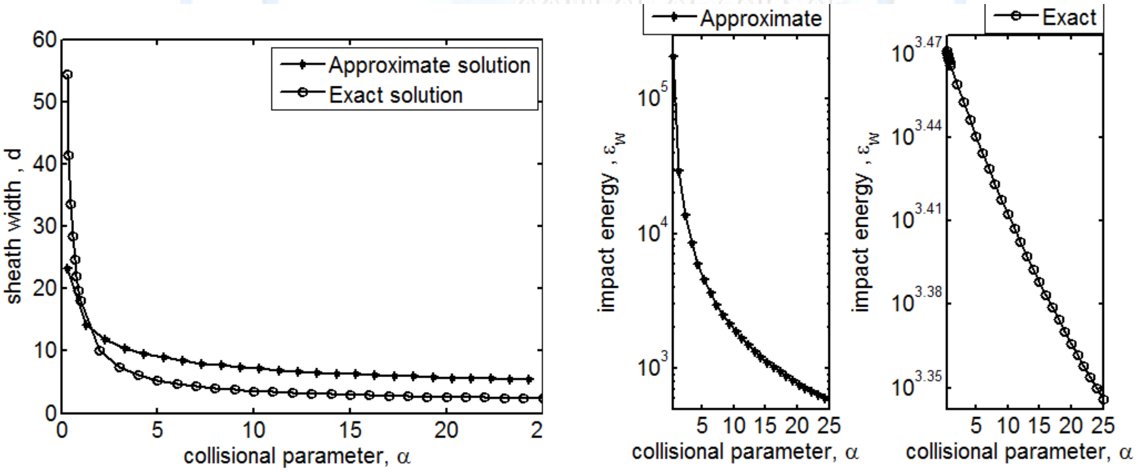

Fig. 5: Comparison between exact and approximate solutions of the dimensionless sheath thickness d and average impact energy as a function of the collision parameter α for constant wall potentials $\eta_w = 10$ in dusty plasma at constant mobility. In the calculations we use charge number $Z = 300$, $\delta = 10$ and $u_0 = 1$.

A Simulation for Studying the Collisional
Dusty Plasma Sheath Phenomenon

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محاكاة حاسوب لدراسة ظاهرة غلاف التصادم في بلازما الغبار

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الخلاصة

أنجزت محاكاة بوساطة الحاسوب لأنموذج رياضي لعمليات التصادم في غلاف بلازما الغبار، حيث يتضمن الأنموذج الرياضي تأثير اعتماد المقطع العرضي لتصادم حبيبات الغبار على قانون أسي للطاقة. وبالاستناد على هذا الأنموذج تم الحصول على الحل العددي المضبوط بدون أي تقريب لمعادلات المتحكمة بغلاف التصادم بوساطة روتينيات طريقة رانج – كوتا للحالة الخاصة عندما تكون تنقلية حبيبات الغبار ثابتة. في هذا البحث تم دراسة تأثير معلمات التصادم على كل من الجهد الكهربائي وعرض الغلاف ومعدل طاقة التصادم عند الجدار لكل من بلازما المتعددة المكونات وبلازما الأيون – الإلكترون عن طريق الحل العددي للمعادلات المميزة. لقد وجد بان اختلاف أعداد الشحنة الكهربائية على حبيبات الغبار والنسبة ما بين الكثافة العددية للأيونات إلى تلك للالكترونات والجهد الكهربائي (وبالتالي المجال الكهربائي) له تأثير قوي على غلاف تصادم بلازما الغبار. أيضا في هذه الدراسة وضعت مقارنة ما بين الحل العددي المضبوط والمعادلات التحليلية التقريبية المشتقة لعرض الغلاف ومعدل طاقة التصادم.