Profile Optimization of the Discharge Electrodes in TEA CO_2 Laser System

Malek A.H. Muhi[*], **Duha S. Ahmed**[**], **Aus A. Najim**[*], **Adawiya J. Haider**[*]

[*] Applied Physics Branch, Applied Sciences department, University of Technology
[**]Nanotechnology advanced material research center, University of Technology

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Abstract

In this work, four profiles (Chang, modified Chang, Ernst 4th order and Ernst 8th order) were considered to fabricate discharge electrodes of a TEA-CO_2 laser from SiC composite. These four profiles were studied and simulated to optimize the distribution of electric field intensity on the electrode surface. As the profile of discharge electrode determines the uniformity of electric field over the whole discharge volume, it was considered experimentally as a reference to choose the best profile. Results show that Ernst 8th order profile is the best among the all simulated and tested profiles. This study may assist the designers of CO_2, Excimer and N_2 gas lasers to optimize the laser system during the designing course by adopting the most efficient profile of the discharge electrodes.

Keywords: Uniform Field Electrode, CO_2 Laser, Gas Discharge, Electrode Profile

امثلية المظهر الجانبي لأقطاب التفريغ الكهربائي لليزر ثنائي اوكسيد الكاربون المستعرض المثار بالضغط الجوي

***مالك عبد الحسن محي، **ضحى سعدي احمد، *اوس عبد الله نجم ، *عدوية جمعة حيدر**

*فرع الفيزياء التطبيقية/قسم العلوم التطبيقية /الجامعة التكنولوجية / بغداد /العراق
**مركز النانوتكنولوجي وبحوث المواد المتقدمة/الجامعة التكنولوجية / بغداد /العراق

الخلاصة

تم في هذا العمل اعتماد اربع مظاهر جانبية (اقطاب نوع تشانغ وتشانغ المعدلة وقطب ارسنت للمرتبة الرابعة الثامنة) لتصنيع اقطاب التفريغ الكهربائي لمنظومة ليزر ثنائي اوكسيد الكاربون ذات التهيج المستعرض عند الضغط الجوي TEA-CO2 Laser من مادة كربيد السيليكون. تم دراسة المظهر الجانبي لهذه الاقطاب ونمذجتها ومن خلال انتظام توزيع كثافة المجال الكهربائي على سطح القطب عمليا تم تحديد افضل مظهر جانبي للاقطاب الاربعة وتم مطابقة هذه النتائج العملية مع النتائج النظرية. اظهرت النتائج ان شكل قطب ارسنت للمرتبة الثامنة هو الافضل من بين النماذج المقاسة والتي قد اختبرت نظريا. ان هذه الدراسة ربما تساعد المصممين لليزرات الغازية (كليزر غاز ثنائي اوكسيد الكاربون و غاز النيتروجين والاكسايمر) في تحديد المظهر الامثل لشكل القطب وتحسينها أثناء التصميم من خلال اعتماد المنظر الجانبي الأكثر كفاءة من أقطاب التفريغ.

الكلمات المفتاحية: قطب المجال الكهربائي المنتظم، ليزر ثنائي اوكسيد الكاربون، غاز التفريغ الكهربائي،المظهر الجانبي للاقطاب

Introduction

The shape of the laser electrodes used in transverse excited (TE) laser has an important effect on the performance of the laser since it affects the distribution and the homogeneity of electric field along discharge region and prevents arcs between laser electrodes. This leads to raise the efficiency of the laser system. As the electric field strength increases at the edges and sharp points, the designer has to make the ends of the electrodes curved as going further from the center. The distance between electrodes increases at the end depending on the geometry of electrode to keep the field distribution uniform. The geometry must satisfy one of the equipotential surfaces and the resulting shape is called **uniform field electrode** (UFE) [1].Achieving the required geometry of electrodes is not simple since their equations cannot be solved analytically. Specially controlled electrodes usually produce very uniform electric field distribution on and between electrode surfaces. Recently, UFE's has received much attention in the production of uniform, large volume, pulsed discharges in TE gas lasers [2-4].

Electrode Design Consideration

The design of UFE's is an old 3D problem and depends on the material to be tested as well as the breakdown field strength. Maxwell [1] studied the electric field distribution between two parallel plates. Rogowski also studied the electric field at insulating and conducting materials [5]. He proposes electrodes for the uniform fields in axially symmetrical system. The profile of electrodes follows the analytical function introduced by Maxwell as [1]:

$$z = \frac{a}{\pi}(w + 1 + e^{w}) \tag{1}$$

$$z = x + iy \tag{2}$$

$$w = u + iv \tag{3}$$

where *v* is the equipotential surface, *u* is line of force, *x* and *y* are spatial coordinates, *a* is separation between two plates, and *z* , *w* are the complex coordinates in *z* and *w* planes,

By substituting:

$$x = \frac{a}{\pi}(u + 1 + e^{u}\cos v) \tag{4}$$

$$y = \frac{a}{\pi}(v + e^{u}\sin v) \tag{5}$$

The profile $\cos v=0$ or $v=|\pi/2|$ is called Rogowski 90^{o} profile.

Chang [6] studied the electric field distribution at the electrode surfaces and he presented an analytical approach to get profiles for finite width UFE's. The profile was derived from the following conformal transformations [6]:

$$z = w + k\sinh(w) \tag{6}$$

For each value of *v*, the profile of corresponding equipotential surface is given by [7]:

$$x = u + k\cos v\sinh u \tag{7}$$

$$y = v + k\sin v\cosh u \tag{8}$$

where *u* is treated as a running variable

The electric field strength for Chang profile is [8]:

$$E^{-2} = \left|\frac{dz}{dw}\right|^2 = (1 + k\cos v\cosh)^2 + (k\sin v\sinh u)^2 \tag{9}$$

The electric field strength (*E*) can also be expressed as a power series expansion in *u* as:

$$E = a_o(v) + a_2(v)u^2 + a_4(v)u^4 + a_6(v)u^6 + \ldots\ldots.. \tag{10}$$

The odd power is missing due to symmetry. To obtain the maximally flat field distribution near the center of electrode (where u=0), the coefficient a_2 has to vanish.

This condition, which is equivalent to $\frac{\partial^2 E}{\partial u^2} = 0$, leads to the following relation between the parameter k and the value of potential function for electrode surface:

$$v_m = arc\cos(-k) = \frac{\pi}{2} + arc\sin(k) \tag{11}$$

The subscript "*m*" indicates that "*v*" is evaluated in the case of maximally flat field distribution near the center of electrode. The previous relation indicates that for best field uniformity is on an equipotential, the value of *v* must be somewhat larger than π/2 (i.e., *k* value must be larger than 0). In this limit, Rogowski and Chang analyses become very similar to each other. On other hand, when the value of *k* increases, Chang's optimized profile deviates further away from Rogowski profile. Chang gave a relation between the parameter k, which determines the geometrical properties of the UFE and aspect ratio of the electrode pair [5]. He defines a parameter $\square_m$ as a maximum fractional variation of electric field that can be tolerated within a critical surface area electrode [7].

$$\delta_m = \left[E(0) - E(u_m)\right] / \ E(0) \tag{12}$$

The value of *x* (or *u*) at the edge would be denoted by x_m or (u_m) and the height of electrode surface at *x*=0 by y_o.

The previous equation can be used to calculate the value of *k*-parameter from the specified values of δ_m and x_m/y_0 using the following equation [7]:

$$\left[\frac{l(1-0.64l)}{1-0.64l-0.36l^2}\right]^6\left[\cosh\left(\frac{x_m}{2y_o}\right)-1\right]^2=\frac{1}{(1-\delta_m)^2} \qquad (13)$$

where $l=\sqrt[3]{k}$

For δ_m<0.1 and the aspect ratio x_m/y_0>0.05 [6], the results of calculation of electric field distribution and electrode profile for different values of k (0.01, 0.06, 0.2) are plotted. In this work, Chang's profile is modified according to the best condition $\cos(\nu)=-k$, which leads to a more compact electrodes profile. It can be shown from Eq. (13) that at $\cos(n)<-k$, $E(u)$ has double maxima located at

$$u=\pm u_d=\mathrm{arccos}h(-\cos\frac{\nu}{k}) \qquad (14)$$

At u=0 where $E(u)$ has local minimum, the factor δ_m is converted to δ_d and given by:

$$\delta_d=[E(u_d)-E(0)]/E(0) \qquad (15)$$

The last equation is used to find the modified value of ν.

Ernst [9-10] studied the electric field distribution at the surface electrodes. He supposed a family of analytic profiles for UFE that have a minimum width and can produce almost high degree of electric field-strength uniformity at the electrode surface. These profiles have analytical expressions as simple as Chang's.

It would be started with the following conformal transformation [9-10]:

$$\varepsilon=w+k_o\sinh w+k_1\sinh 2w+\ldots\ldots \qquad (16)$$

where $\varepsilon=x+iy$ and $w=x+iy$.

For each value of ν ($|\nu|<\pi$), the profile of the corresponding equipotential surface is given by:

$$x=u+k_o\sinh(u)\cos(\nu)+k_1\sinh(2u)\cos(2\nu) \qquad (17)$$

$$y = v + k_o \cosh(u)\sin(v) + k_1 \cosh(2u)\sin(2v) \tag{18}$$

This profile is symmetric with respect to *y*-axis and the equipotential surfaces ($-v$, $+v$) are mirror image with respect to the *x*-axis, which are pre-requisites for a UFE.

From equations (15) and (16), it can be seen that profiles are not uniquely determined. Three independent variables k_o, k_1 and v, determine the form of the profile as well as the electric-field strength distribution over the electrode surface.

To find the optimum profile, an expression is needed for the electric-field strength as [11]:

$$E^{-2} = \left|\frac{d\varepsilon}{dw}\right|^2 = \left|1 + k_o \cosh w + 2k_1\, osh 2w\right|^2 = f^2(u) + g^2(u) \tag{19}$$

where

$$f(u) = 1 + k_o \cosh w \cos v + 2k_1 \cosh 2u \cos 2v \tag{20}$$

$$g(u) = k_o \sinh u \sin v + 2k_1 \sinh 2u \sin 2v \tag{21}$$

The power series expansion can be used to solve the electric field equations, many methods have been used. When the electric field strength is expressed as a power-series expansion around u=0:

$$E = E_o(k_o, k_1, v) + E_2(k_o, k_1, v)u^2 + E_4(k_o, k_1, v)u^4 + \dots\dots\dots \tag{22}$$

This expression is expanded to fourth degree with absence of odd power due to symmetry. According to degree of expansion (the exponent of variable *u*), the profile is called Ernst 4th order profile and expressed as “Ernst 4th order profile”. The expression of electric field strength as a power series around the center of the electrodes (u=0 or x=0) would be used to optimize the calculation.

Solving the power series at x=0 or u=0 and finding the optimum profile (optimum value of k_1 and v) would require the lower coefficients E_2, E_4 to vanish. This condition is equivalent to $\partial^2 E/du^2$ and $\partial^4 E/du^4$ and leads to [12-13]:

$$E_2 = -[f(0)\,f^{(2)}(0) + (g^{(1)}(0))^2]/\,f^3(0) = 0 \qquad (23)$$

$$E_4 = -[f(0)\,f^4(0) + 3(f^{(2)}(0))^2 + 4g^{(1)}(0)\,g^{(3)}(0)]/\,f^3(0) = 0 \qquad (24)$$

The exponent between the brackets denotes to the number of differentiation with respect to *u*.

Ernst also proposes that adding extra terms to the same conformal transformation may lead to improve the profile. The conformal transformation would take the form:

$$\varepsilon = w + k_o \sinh w + k_1 \sinh 2w + k_2 \sinh 3w....... \qquad (25)$$

As in Eq. (13), we have:

$$x = u + k_o \sinh(u)\cos(v) + k_1 \sinh(2u)\cos(2v) + k_2 \sinh(3u)\cos(3v) \qquad (26)$$

$$y = v + k_o \cosh(u)\sin(v) + k_1 \cosh(2u)\sin(2v) + k_2 \cosh(3u)\sin(3v) \qquad (27)$$

To find the optimum profile, an expression is needed for the electric-field strength [10]:

$$\begin{aligned} E^{-2} = \left|\frac{d\varepsilon}{dw}\right|^2 &= \left|1 + k_o \cosh(w) + 2k_1 \cosh(2w) + 3k_2 \cosh(3w)\right|^2 \\ &= f^2(u) + g^2(u) \end{aligned} \qquad (28)$$

Since

$$f(u) = 1 + k_o \cosh u \cos v + 2k_1 \cosh 2u \cos 2v + 3k_2 \cosh 3u \cos 3v \qquad (29)$$

$$g(u) = k_o \sinh u \sin v + 2k_1 \sinh 2u \sin 2v + 3k_2 \sinh 3u \;\sin 3v \qquad (30)$$

When the electric field strength is expressed as a power-series expansion around *u*=0 as:

$$\begin{aligned} E = {} & E_o(k_o, k_1, k_2, v) + E_2(k_o, k_1, k_2, v)u^2 + E_4(k_o, k_1, k_2, v)u^4 \\ & + E_6(k_o, k_1, k_2, v)u^6 + ... \end{aligned} \qquad (31)$$

The parameters k_o, k_1, k_2 and *v* here are used to optimize the profile of electrodes and optimize the field strength distribution by requiring the three coefficients E_2, E_4 and E_6 from Eq. (21a) to vanish, where E_2 and E_4 are the same as Ernst 4th profile while E_6 is given by:

$$E_6 = -[f(0)\,f^{(6)}(0) + 15\,f^{(2)}(0)\,f^{(4)}(0) + 6\,g^{(1)}(0)\,g^{(5)}(0) + 10\,g^{(3)2}(0)]/\,f^{3}(0) \\ = 0 \qquad (32)$$

Experimental Work

Before starting to fabricate the electrodes, a simulation study has been applied to choose the best profile using computer software to show different profiles. The equations of electrode profiles; Chang, Ernst 4th and Ernst 8**th** order, were used in this study. The laser electrodes were fabricated from SiC composite and copper powders and Fig. (1) shows the UFE's fabricated in different four profiles.

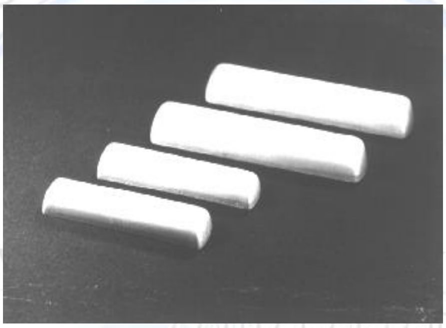


Fig (1): The fabricated electrodes in this work

Many aspects, such as laser head dimensions and output laser pulse energy, influence the design of laser electrodes. Chang [6] approximates the laser electrodes profile with equipotential surface given by equations (7) and (8). He considered the following conditions $x>0$, $y>0$ (i.e. first quadrant) and $v\approx\pi/2$, to simplify the calculation. Using the electric field strength given by equation (9) combined with equation (10), the definition of the parameter δ_m (equation (11)) and the relation between k, δ_m and v given by equation (12), one the electrode profile . Computer simulation with MATLAB and Maple software was used to solve these equations and find the desired electrode profile and the electric field distribution on the electrode. Figure (2) shows the flow chart of the calculation method according to Chang’s approximation.

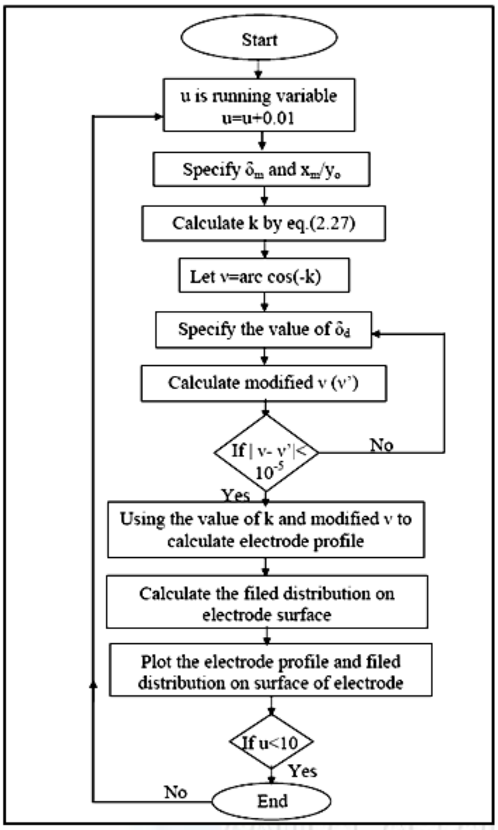


Fig (2): Flow chart of Chang design program steps

Figure (3) shows the flow chart of the simulation program used for solving the set of equations of E_2 and E_4 to find values of k_1 and v. It is clear that the optimized value of v deviates very slightly from π/2 up to k_o values of 0.1. So, for all practical profiles, the v value can be, therefore, approximated by π/2. The value of k_1 can then be found from equation (23) with v= π/2 yielding:

$$k_0^2 - 8k_1(1-2k_1) = 0 \tag{33}$$

or

$$k_1 = \frac{1}{4} - \frac{1}{4}(1-k_0^2)^{\frac{1}{2}} \tag{34}$$

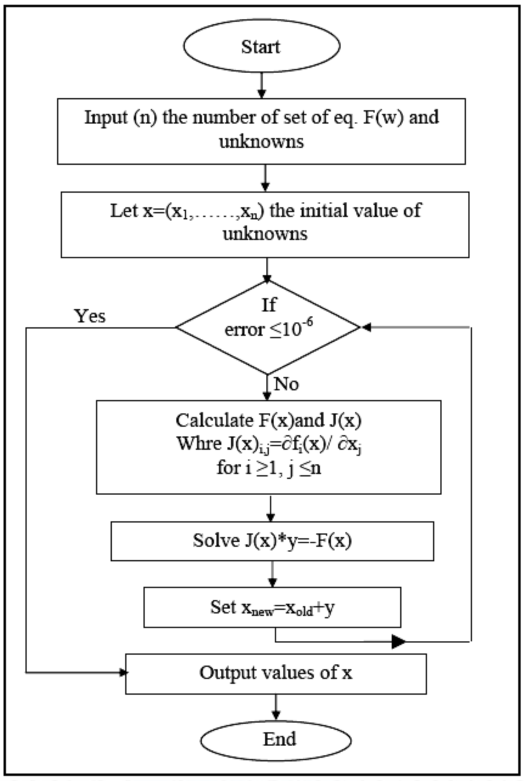


Fig (3): Flow chart of simulation program for Ernst profile

In Ernst 8th order profile, four independent variables k_o, k_1, k_2 and v determine the electric field distribution and electrode profile [14 and15]. Once k_o has been chosen as an independent variable, the variables k_1, k_2 and v can be used to optimize the electric field strength distribution and electrode profile.

Now, all three coefficients E_2, E_4 and E_6 from equation (31) are required to vanish in order to solve equations (23), (24) and (32). It seems that the value of v does not deviate greatly from π/2. It can be assumed that $v=\pi/2$ and the optimized coefficients k_1, k_2 can be found by solving the cubic equation and get the following roots:

$$k_1 = \frac{1}{4}\{1-[1-(1-k_0-9k_2)^2]^{\frac{1}{2}}\} \tag{35}$$

$$k_2 = \frac{5}{81}k_0\{1-[1-\frac{9}{25}(1-\frac{8k_1}{k_0}+\frac{64k_1^2}{k_0^2})]^{\frac{1}{2}}\} \tag{36}$$

Results and Discussion

1. Modification of Chang's Profile for Laser Electrodes

Figure (4) shows a comparison of the profiles between Chang and modified Chang profiles. It is noted that for the Chang profile, when the value of *k* increases, the electrode profile becomes compact while when *k* value decreases the electrode profile is enlarged. The modified Chang profile has more compactness than Chang's. Figures (5) and (6) show the electric field distribution for Chang and modified Chang electrodes. It appears clear that the modified Chang electrodes give more acceptable results than Chang's, and the value of δ_d=0.009 gives the more suitable results.

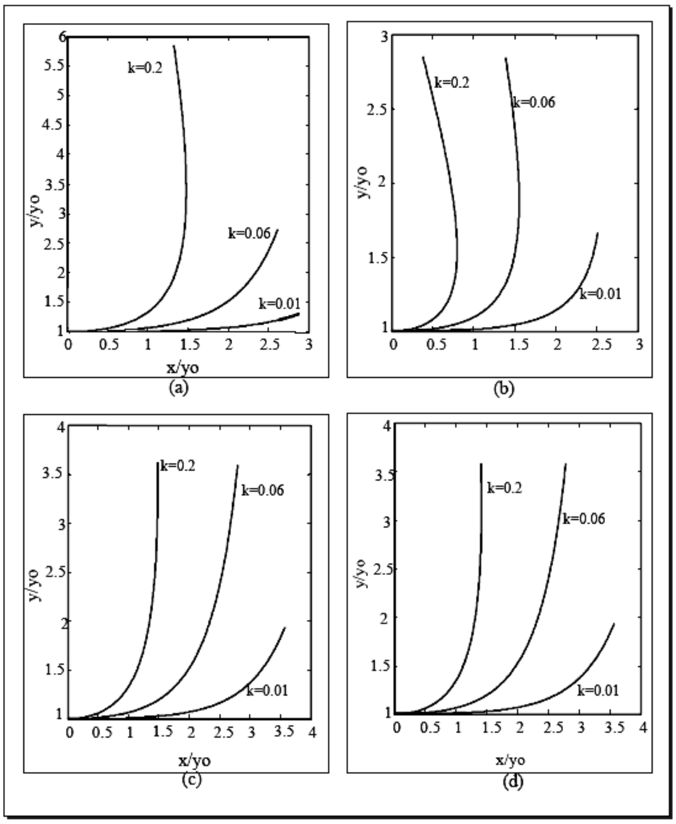


Fig (4): Electrode profile for different k values, (a) Chang (b) modified Chang at δ_d=0.009 (c) modified Chang at δ_d=0.006 (d) δ_d=0.001

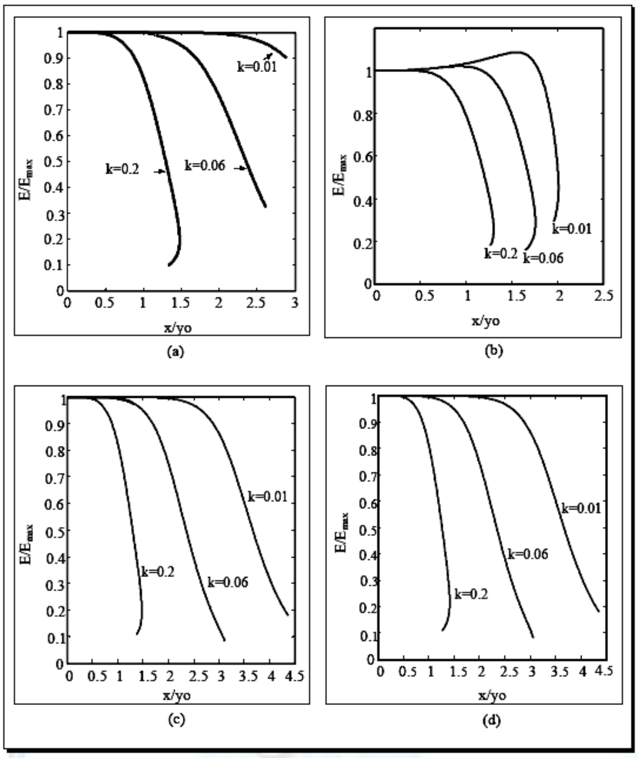
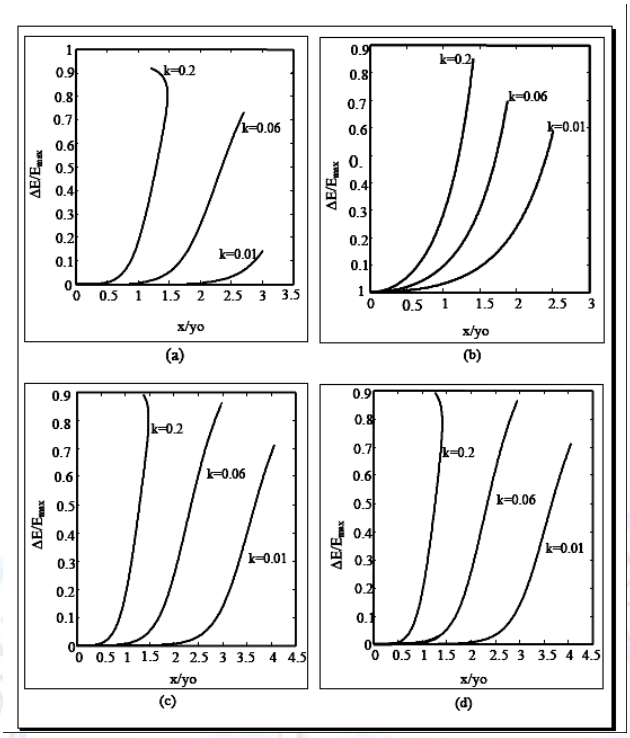


Fig (5): Electric field strength normalized to maximally flat point for different *k* values, (a) Chang (b) modified Chang at δ_d=0.009 (c) modified Chang at δ_d=0.006 (d) modified Chang at δ_d=0.001



Fig(6): Electric field strength distribution for different *k* values (a) Chang (b) modified Chang at δ_d=0.009 (c) modified Change at δ_d=0.006 (d) modified Chang at δ_d=0.001

2. The Best Shape of Electrode

Figure (7) shows the profiles of Chang, Ernst 4th and Ernst 8th electrodes for different k_o values. From this figure, it is noted that the Ernst 8th profile is the more compact than the two others since its smaller dimension by (10-20)%. With different value of k_o, there is a significant difference in Ernst 8th profile. The surface area of the electrode is decreased due to the reduction in both width and thickness and hence the inductance would decrease too. This definitely affects the width of the output laser pulse.

Figure (7c) shows that Ernst 8th profile is similar to Chang's profile till x/y_o is equal to 0.85 at which the behavior is changed and the profile becomes more compact and largely curved. Figure (7d) shows the comparison between the modified Chang, Ernst 4th and Ernst 8th profiles for k_o value of 0.2 and three different δ_d values (0.001, 0.006 and 0.009). It appears clear that Ernst 8th profile has also the more acceptable results.

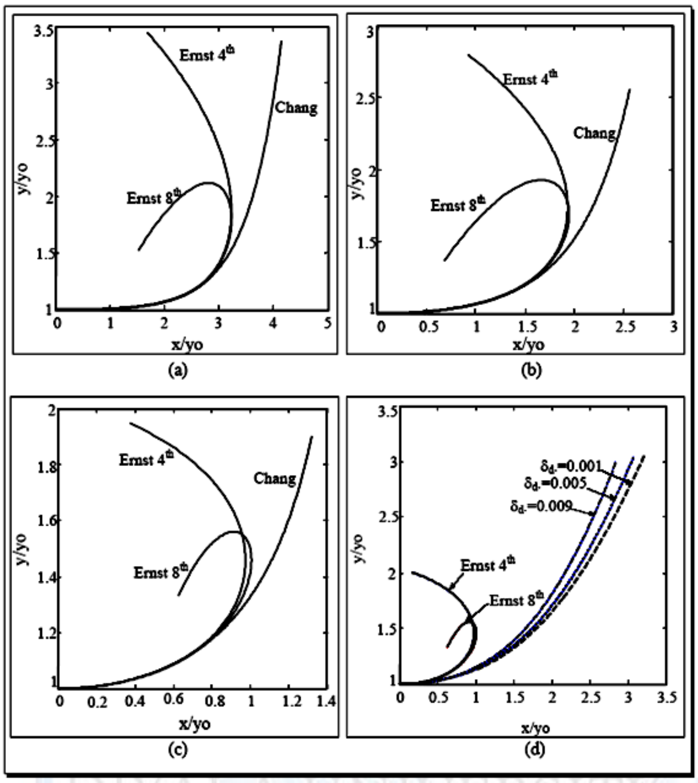
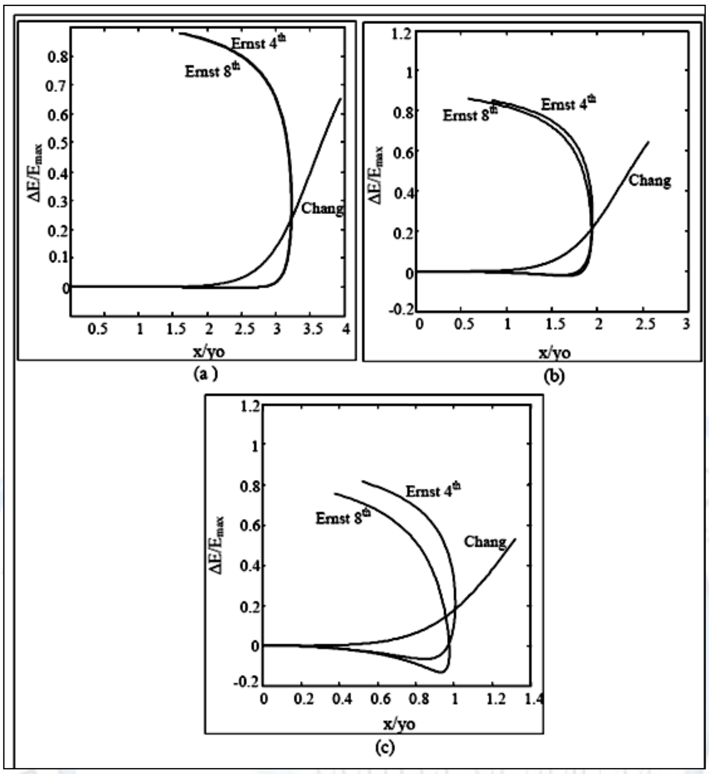


Fig (7): Profiles of Chang and Ernst electrodes with k_o (a) 0.01 (b) 0.06 (c) 0.2 (d) k_o=0.2 and modified Chang

3. Distribution of Electric Field on the Electrode Surface

It is found that the profile of Ernst 8th order electrode has the more uniform electric field distribution. The uniformity can be explained with respect to the values of E/E_{max} , $\Delta E/E_{max}$, where E represents the value of electric field strength on the surface of electrode, E_{max} represents the electric field strength at the maximally flat point (x=0) of the electrode surface with respect to x-axis and after which the electrode begins to be curved, and ΔE represents the difference in electric field at the maximally flat point and electric field strength at any other points.

Figure (8) shows the $\Delta E/E_{max}$ for three electrode profiles (Chang, Ernst 4th and Ernst 8th). It shows that Ernst 8th order electrode has the acceptable results as the $\Delta E/E_{max}$ curve . Each one of the three figures was retreated before the curve of the other two profiles, and this behavior appears clear in Figure (8c). This would support the profile shown in Figure (7), since Ernst 8th order profile is the more compact.



Fig(8): Electric field strength distribution in terms of $\Delta E/E_{max}$ for Chang and Ernst profile with different *k* values (a) 0.01 (b) 0.06 (c) 0.2

Figure (9) shows the E/E_{max} for the profiles of three electrodes (Chang, Ernst 4th order and Ernst 8th order). It can be seen that Ernst 8th order profile gives more acceptable results than the two others. This result is in agreement with the compactness of the Ernst 8th order electrodes. Figures (10) and (11) show a comparison between $\Delta E/E_{max}$ and E/E_{max}, respectively, for modified Change, Ernst 4th order and Ernst 8th order electrode profiles. It appears that Ernst 8th order profile still has the more acceptable results than modified Chang profile.

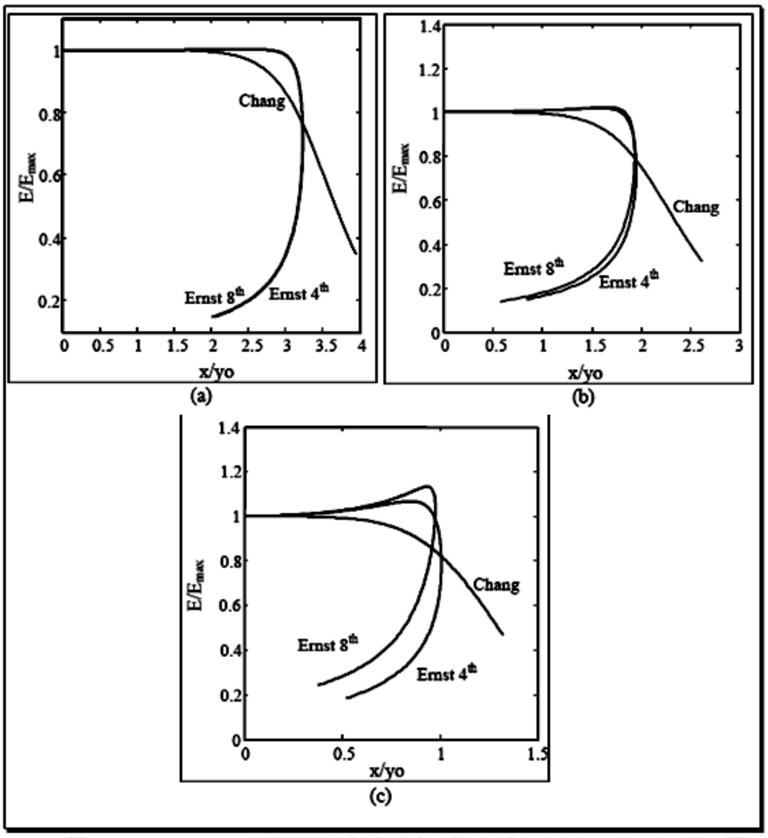
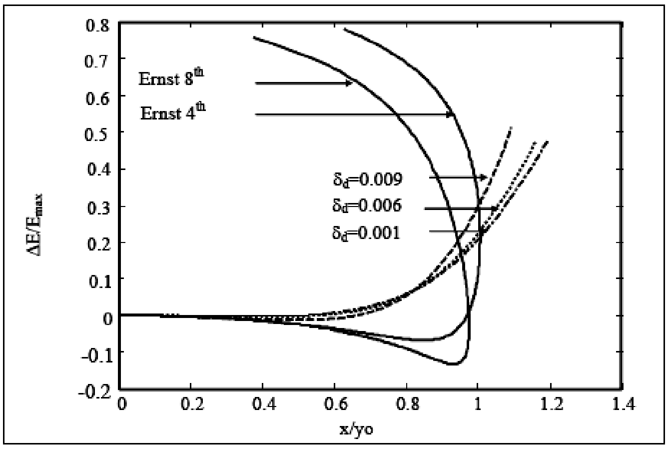


Fig (9): The electric field strength normalized to maximally flat point of Ernst and Chang electrodes at different k_o (a) 0.01 (b) 0.06 (c) 0.2



Fig(10): Electric field strength in terms of $\Delta E/E_{max}$ for modified Chang and Ernst profiles

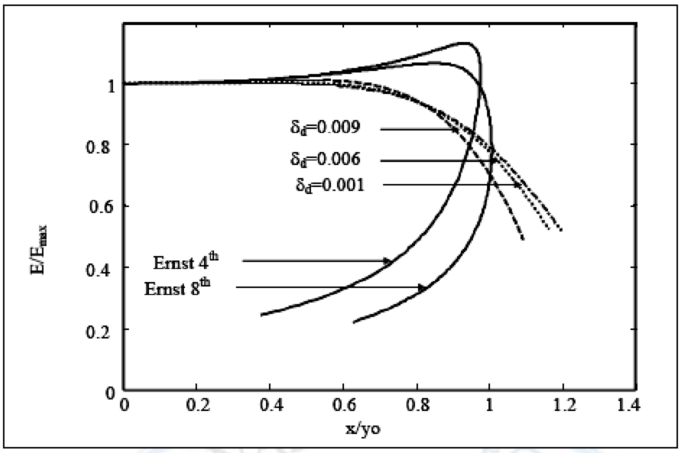


Fig (11): Electric field strength in terms of *E/E_{max}* for modified Chang and Ernst profiles

Conclusion

According to the obtained results in this work, the electric field distribution in a discharge volume is mainly affected by the profile of the discharge electrode and the optimum profile to produce uniform and homogeneous discharge is Ernst 8th order, as proved by the comparison with other profiles such as Chang, modified Chang and Ernst 4th order. In order to optimize the characteristics of output laser pulse, the inductance of the electrode is required to be reduced as much as possible. This can be achieved by the compact electrode profile. The optimization of the electrode profile has been confirmed by the experimental results through employing such electrodes in a home-built TEA-CO_2 laser system.

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