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Studying the Stress Analysis in Leaf Spring by Finite Elements Method

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ABSTRACT - Light truck is considerably crucial for all the world economic growth since it was used in transportation for various kinds of commodities. Leaf spring is the most admire alternative utilized in the suspension system of this specific truck due to their low production cost, reliability and maintainability. In this, study a rational technique to design leaf spring for light truck is investigated. Simulation resembles to constrained condition when mounting on the vehicles were undertaken to examine the stresses distribution for each leaf. In this research the leaf spring is used on light truck with following information: Number of leaves equal five, Thickness of each leaf equal 10mm, length of distance from vertical line that divides the leaf spring to two equal parts, to one end of side of leaf spring, varies from (500 to 1000) mm, Radius of bow- shaped leaf spring varies from (1250 to 3000) mm and constant load applied on the first leaf equal 1000N on each side. The number of cases studied equal 24 case. The finite element method was applied as a method of analysis to examine the stress distribution for each leaf.

***Keywords:* -** Stress leaf spring, Finite element.

1- INTRODUCTION

In order to provide satisfactory comfort, shocks from bumps in the road surface must not be transmitted to the cab and superstructure. Springs are therefore provided between wheels and the chassis frame.

Typically, leaf spring design find their greatest application within automotive industry. Most leaf spring applications are comprised of multi-leaf system found primary in rear suspensions.

Recently, automotive industry requires higher level of design and calculation almost in every part in both fabrication and testing which make it possible to improve and develop products.

Currently, the design of multi-leaf spring and the prediction of behaviors are more efficient when using finite elements method. The purposes of this research are to analyze, develop and validate finite element models of multi-leaf spring.

2- THEORETICAL PART (BASIC THEORY)

Although, finite elements analysis will determine the behaviors of contact verse non-contact, it is important to capture the basic fundamental Multi-leaf spring analysis relies on basic theory. Where each leaf in the system can be idealized in to a diamond shape relative to the leaf spring width and length as shown in Fig(1). The theory ties load to displacement relative to the elastic stiffness of the spring system"1".

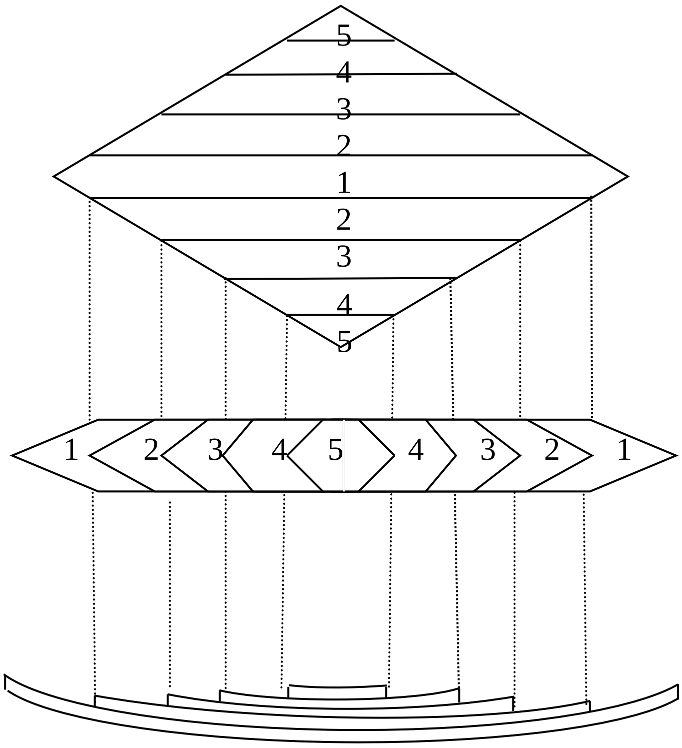


Fig. (1):- Leaf Spring Idealized Diamond Form

2-1- Deflection

The resulting deflection from simple beam theory"2"

R= EI/M …..for leaf spring.

$\delta = 3PL^3/(8E\ \mathbf{n}*b*t^3)$ …..(1) *where*

R= Radius of curvature(mm)	E= Elastic Modules KPS	I= Moment of Inertia	M= Applied Moment N.mm
δ= Deflection (mm)	P= Load (N)	L= Length of Leaf (mm)	n= Number of Leave
b= Width of Leaf (mm)	t = Thickness of leaf(mm)		

2-2- Spring Rate

If load is written in terms of deflection, P= f(δ) then the slope of the curve defines the spring rate:

K= P/ δ

where ..(k = spring stiffness)

2-3- Finite Element Analysis

In this research, the analysis of multi-leaf spring were performed using finite elements method for determining stress distribution with following assumptions:

- The material properties are homogeneous.
- The effect of residual stresses such as heat treatment stress peeing, or center clamp are omitted.
- Spring shackle are not modeled.
- No inter leaf friction is considered.
- Only vehicle and payload were applied vertically.

The classical finite elements method are used with modification where all points (nodes) from points of contact, (between any two nodes on contact leaves) regarded to have the same displacement and the same internal forces. Dependence upon the Second Newton's Law Σ F= m* a, but at points of contact (a)=zero because there is no internal motion between any two contact leaves of spring, this led's to Σ F= 0, where (a) is the acceleration and (m) is the mass**.** This can be considered between first and the second leaf also can be considered between any two contact leaves of spring "3" "4". Where

$U_{nX1}= U_{nX2}$ and $U_{nY1}= U_{nY2}$,,, $U_{nX2}= U_{nX3}$ and $U_{nY2}= U_{nY3}$

$F_{nX1}= F_{nX2}$ and $F_{nY1}= F_{nY2}$,,, $F_{nX2}= F_{nX3}$ and $F_{nY2}= F_{nY3}$

Where

(U) displacement

(n) number of nodes

(x, y) x-axis and y-axis respectively

(F) contact force

1,2,3 symbol of body.

The calculation of stress by using finite elements method(**plan strain)** is, as shown in equation (2)

$$\underbrace{\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}}_{\text{stress column}} = \underbrace{\{E/(1+\nu)(1-2\nu)\}\begin{Bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & (1-2\nu)/2 \end{Bmatrix}}_{\text{matrix of (plane strain)}} \underbrace{\begin{Bmatrix} dN_1/dx & 0 & dN_2/dx & 0\ldots \\ 0 & dN_1/dy & 0 & dN_2/dy\ldots \\ dN_1/dy & dN_1/dx & dN_2/dy & dN_2/dx.. \end{Bmatrix}}_{\text{matrix of shape function}} \begin{Bmatrix} \delta_x \\ \delta_y \\ \gamma_{xy} \end{Bmatrix} \quad \ldots..(2)$$

where σ_x , σ_y , τ_{xy} are Horizontal Stress, Vertical Stress and Shear Stress respectively. (E) Modules of Elasticity.

(ν) Poisson's Ratio.

(δ_x) Horizontal Displacement.

(δ_y) Vertical Displacement.

$\gamma_{xy} = \delta_x + \delta_y$.

$N_i(x, y)$ is Shape Function.

3- APPLICATION PART

3-1- Mechanical Properties

For all analysis, leave spring were made of steel ASTM A27-55, grade 70-36, therefore, the mechanical properties in the finite element models are present in Table (1)

Table (1):- Mechanical properties of steel ASTM A27-55 "5"

Mass Density (Kg/m^3)	Modulus of Elasticity (GPa)	(*v*) Poisson's Ratio	Tensile Strength Ultimate (MPa)	Tensile Strength Yield (MPa)
7850	21	0.3	1170	1070

3-2- Geometry

For all analysis the shape of leaf springs with main dimensions are presented in Fig.(2)

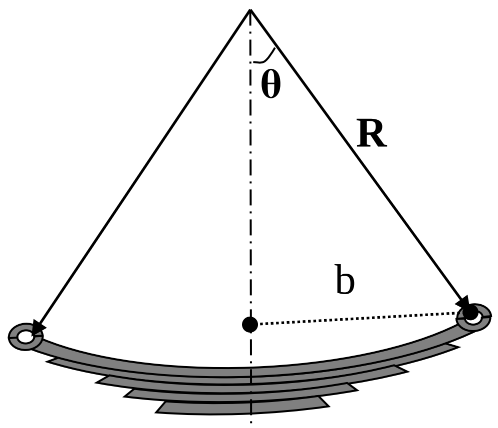


Fig. (2):- Indicates the shape of leaf spring with major dimensions
R= Radius of curvature, θ= angle of curvature and , b= length of distance from vertical line that divides the leaf spring to two equal parts , to one end side of leaf spring

The relationship between (R) and **(θ)** for different values of (b) are presented in Fig.(3). In this research the following characteristics of multi-leaf spring that taken in consideration for light trucks are:

Length (b)= 1000mm , 639mm and 500mm.

Number of leaves = 5.

Radius (R) varies from (1250~ 3000)mm.

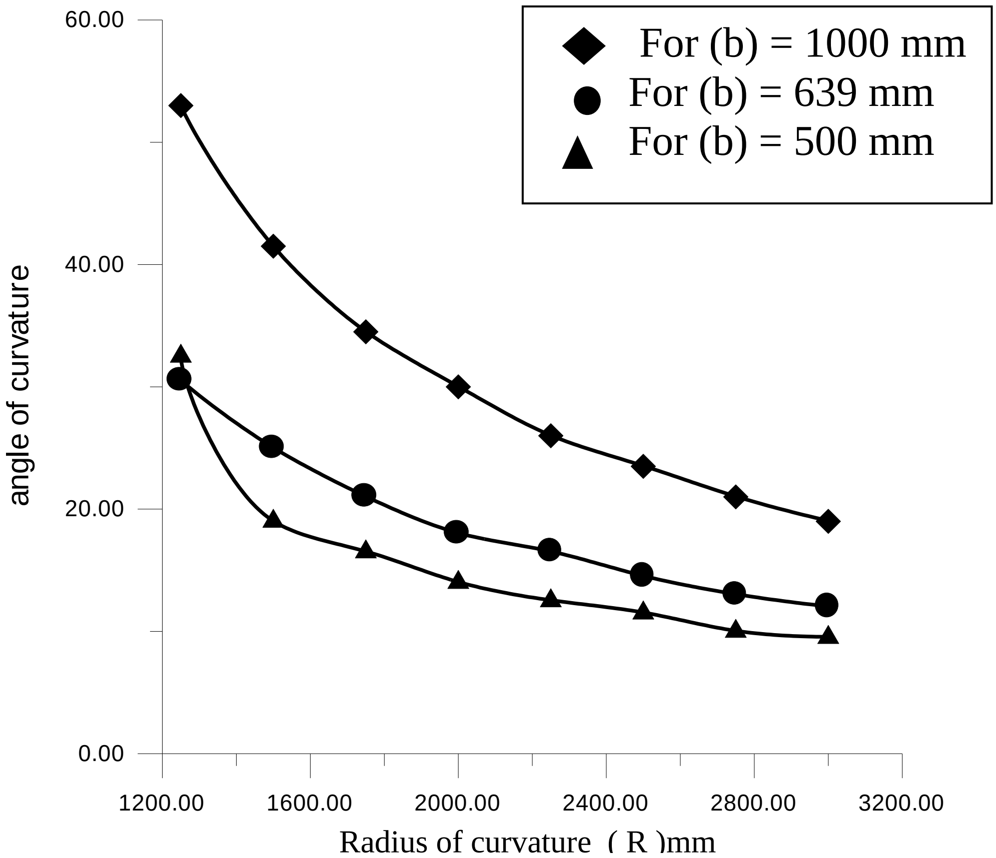


Fig. (3):- indicates the relation between radius of curvature and half of angle of curvature for leaf spring

3-3- Load of Multi - Leaf Spring

Stress analysis of multi – leaf spring were considered when the spring is subjected to maximum load at 1000N to determine the stress distribution and critical region of high stress which could cause failure to five leaves "7".

3-4- Application of Finite Element Method in Multi - Leaf Spring

Depending upon section 2.3 & 3.2 the cases to be studied are (24) cases with ninety-four nodes considered on each case. One of these cases is shown in appendix (a).

Appendix (a) shows that there are three nodes considered as fixed nodes and two nodes considered as loading nodes. Also (48) nodes are considered as contact nodes between any two leaves and the plane of analysis is plane strain.

3-5- Stress Analysis of Multi - Leaf Spring

To represents Von Misses stress distribution on each leaf, Table (2) indicates the stress distribution in upper and lower sides of five leaves for one case with dimensions (b=500mm,R=500mm and $\theta = 28^{o}$) as an example.

It was found that on the first lower side of leaf is the most dangerous region which give the maximum stresses. Tables (3),(4) and (5) indicates these maximum stresses for (b) = 1000mm , 639mm and 500mm. with different value of radii of curvature (R) = 1250 to 3000mm.

4- DISCUSSION

Stress analysis of multi leaf spring were considered, when the spring is subjected to maximum load of 1000N to determine the stress distribution and critical region of high stresses which could cause failure to the spring. Von Mises stresses on each leaf from first to fifth were calculated for different radii of curvature. Three groups are considered for light truck in this study, each group represent different value of (b) i.e.

First group b=1000mm. Second group b=639mm. Third group b=500mm.

In all above groups the radii of curvature varies from 1250mm to 3000mm. The relationship between radius of curvature(R) and angle of curvature(θ) are inversely proportional as shown in Fig(3),this is because when distance (b) remain constant then the radius of curvature increased when the angle of curvature decreasing depending on equation
$\theta = \tan^{-1}(b/R)$.

On each leaf it is noted that the leaf have two sides, the upper side at internal curvature and the lower side at external curvature. In all (24) cases it was found that the stresses at lower side of leaves is more than the upper sides for the first four leaves as shown in fig(4) and Table (2), this is due to arc geometry of leaves and they are active, while the fifth leaf is stationary and may be called as helper spring.

The first four leaves gradually engages and complete contact is engaged when the spring operated at the constant high rate designed load.

If the curvature beam is free there is a bending stress at lower side of curvature beam which has the same stress as on upper side of curvature beam with different signs. But in this research the upper and lower sides of curvature beam are not free since there are contacts between the points on two adjacent leaves. This contact causing different force distribution on two sides upper and lower surfaces of leaf spring as shown in Fig.(5).

It can be seen that the stresses on the lower surface are bending stress combined with compression stress and they are in the same direction which make the stresses at lower side greater than that on upper surface which has bending stress combined with tensile stress. Then the deflection in lower side will be greater than that on upper side.

Also Table (2) indicated that there is compression stress on lower side and the forces signs are negative (-). But if the contact forces between two leaves on the upper side then the sign of contact forces are positive (+), which reducing the compression stress at this side.

Fig(4) shows that the stress distribution on lower and upper sides looks like sine wave and this result is approximately the same as in reference [8],this is due two (+)sign at upper side and (-) sign at lower side in Table(2) and Fig.(5).

The arc geometry for leaf spring gives the same behavior between any two contact points, so when one point goes up then the other goes down and so on for any other contact points.

The first leaf has many nodes and have high stresses for different values of radii of curvature in (24) cases. Fig.(6) shows that in case of b=1000mm the maximum stress is decreasing when the radii of curvature is increasing, this is due to limit factor for stress magnitude for arc curvature.

Also Fig.(6) shows that with the same value of radius of curvature the stresses increased with increased length of leaf.

5- CONCLUSION

1. The magnitude of stress in lower side surface for all leaves of spring is more than stress in upper side surface.

2. The maximum stresses are equals for all leaves except that on the last leaf (where last leaf is fixed).
3. The stress distribution for first four leaves is us sine wave.
4. The number of nodes of maximum stress in first leaf is more than any other leaves.
5. The effect of radius of curvature on the stress is greater in leaf spring which has large value of (b) than that has small value of (b).

Table (2):- indicates the stress and internal distributions in upper and lower five leaves spring according to appendix (a) where radius of curvature R= 2000 mm , angle of curvature $\theta = 28^o$ and distance b=500mm

Number of node	Stress N/mm^2	Internal force (N)	Number of node	Stress (N/mm)	Internal force (N)
25 upper leaf num. (1)	9.5	0	60 lower leaf num. (2)	169	-1635
26 upper leaf num. (1)	23	0	61 lower leaf num. (2)	32	1583
27 upper leaf num. (1)	35	0	62 lower leaf num. (2)	166	-1612
28 upper leaf num. (1)	25	0	63 lower leaf num. (2)	33	1854
29 upper leaf num. (1)	35	0	64 lower leaf num. (2)	138	0
30 upper leaf num. (1)	24	0	65 upper leaf num. (3)	19	-1869
31 upper leaf num. (1)	35	0	66 upper leaf num. (3)	138	1625
32 upper leaf num. (1)	26	0	67 upper leaf num. (3)	43	-1566
33 upper leaf num. (1)	36	0	68 upper leaf num. (3)	33	1635
34 upper leaf num. (1)	29	0	69 upper leaf num. (3)	42	-1583
35 upper leaf num. (1)	20	0	70 upper leaf num. (3)	38	1612
36 lower leaf num. (1)	53	0	71 upper leaf num. (3)	26	-1854
37 lower leaf num. (1)	25	1960	72 lower leaf num. (3)	125	0
38 lower leaf num. (1)	151	-2011	73 lower leaf num. (3)	34	2081
39 lower leaf num. (1)	30	2093	74 lower leaf num. (3)	165	-1799

40 lower leaf num. (1)	165	-2027	75 lower leaf num. (3)	29	1419
41 lower leaf num. (1)	29	1970	76 lower leaf num. (3)	165	-1768
42 lower leaf num. (1)	160	-2045	77 lower leaf num. (3)	33	2066
43 lower leaf num. (1)	30	2129	78 lower leaf num. (3)	137	0
44 lower leaf num. (1)	154	-2032	79 upper leaf num. (4)	20	-2081
45 lower leaf num. (1)	25	1962	80 upper leaf num. (4)	43	1799
46 lower leaf num. (1)	54	0	81 upper leaf num. (4)	58	-1419
47 upper leaf num. (2)	22	-1960	82 upper leaf num. (4)	42	1768
48 upper leaf num. (2)	40	2011	83 upper leaf num. (4)	27	-2066
49 upper leaf num. (2)	34	-2093	84 lower leaf num. (4)	149	0
50 upper leaf num. (2)	38	2027	85 lower leaf num. (4)	38	2350
51 upper leaf num. (2)	44	-1970	86 lower leaf num. (4)	205	-2701
52 upper leaf num. (2)	38	2045	87 lower leaf num. (4)	38	2350
53 upper leaf num. (2)	44	-2129	88 lower leaf num. (4)	163	0
54 upper leaf num. (2)	40	2032	89 upper leaf num. (5)	1.9	-2350
55 upper leaf num. (2)	30	-1962	90 upper leaf num. (5)	22	2701
56 lower leaf num. (2)	125	0	91 upper leaf num. (5)	1.8	-2350
57 lower leaf num. (2)	33	1869	92 lower leaf num. (5)	12	876.91
58 lower leaf num. (2)	165	-1625	93 lower leaf num. (5)	9	245.86
59 lower leaf num. (2)	32	1566	94 lower leaf num. (5)	12	876.72

Table (3):- indicates the stress distribution in first lower leaf according to appendix (a) with many magnitude of radii of curvature for leaf spring where b=1000 mm

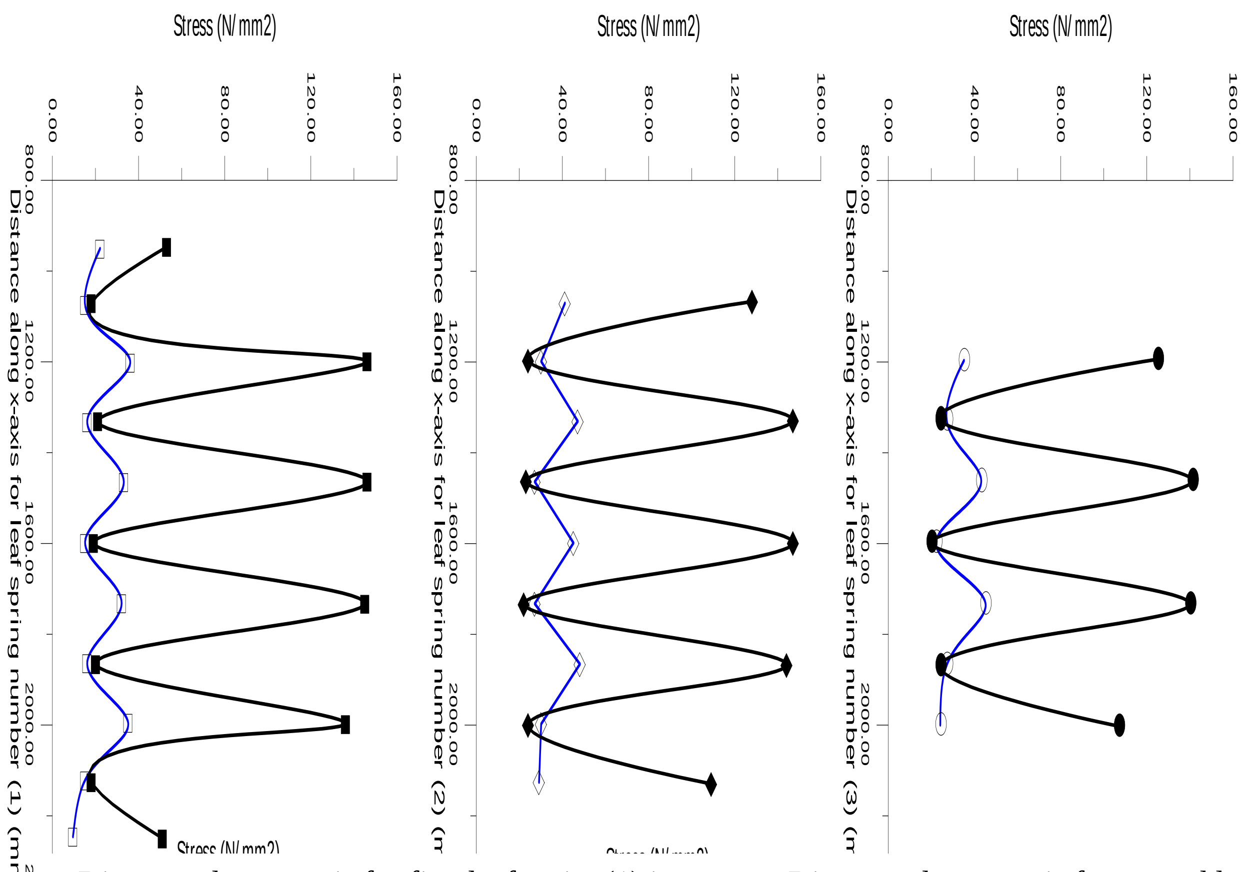
Radius Mm	Stress in Node 36 N/m m^2	Stress in Node 37 N/m m^2	Stress in Node 38 N/m m^2	Stress in Node 39 N/m m^2	Stress in Node 40 N/m m^2	Stress in Node 41 N/m m^2	Stress in Node 42 N/m m^2	Stress in Node 43 N/m m^2	Stress in Node 44 N/m m^2	Stress in Node 45 N/m m^2	Stress in Node 46 N/m m^2
1250	78	10	232	9	220	8	227	10	255	10	103
1500	53	11	158	10	156	10	159	11	169	11	68
1750	46	11	135	11	136	10	138	12	142	11	57
2000	43	11	121	12	124	11	125	12	127	11	51
2250	41	11	111	12	118	12	119	13	116	11	46
2500	39	11	107	13	111	12	112	13	111	11	43
2750	38	12	102	13	110	13	110	13	106	12	41
3000	37	12	98	13	166	13	107	14	101	12	39

Table (4):- indicates the stress distribution in first lower leaf according to appendix (a) with many magnitude of radii of curvature for leaf spring where b=639 mm

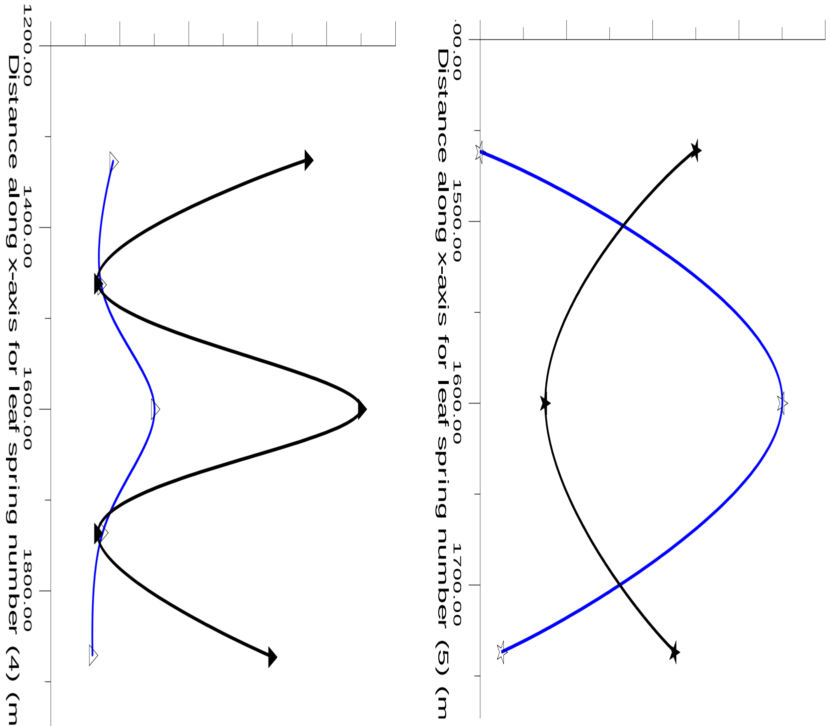
Radius Mm	Stress in Node 36 N/m m^2	Stress in Node 37 N/m m^2	Stress in Node 38 N/m m^2	Stress in Node 39 N/m m^2	Stress in Node 40 N/m m^2	Stress in Node 41 N/m m^2	Stress in Node 42 N/m m^2	Stress in Node 43 N/m m^2	Stress in Node 44 N/m m^2	Stress in Node 45 N/m m^2	Stress in Node 46 N/m m^2
1250	53	18	147	19	151	18	153	20	151	18	58
1500	51	18	138	20	144	19	145	21	142	18	53
1750	50	19	134	21	142	20	143	22	137	19	51
2000	50	19	13	22	140	21	141	23	133	19	49
2250	48	19	126	22	135	21	136	22	128	19	47
2500	48	19	124	23	135	22	136	23	127	19	46
2750	48	20	124	23	135	23	136	23	126	20	46
3000	48	20	122	23	135	23	136	24	124	19	45

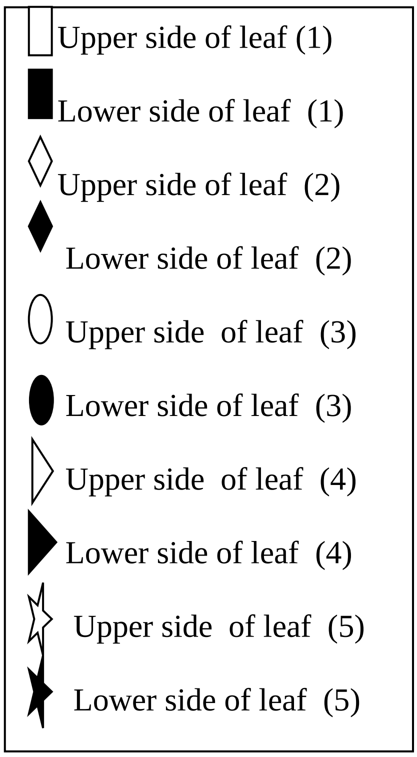
Table (5):- indicates the stress distribution in first lower leaf according to appendix (a) with many magnitude of radii of curvature for leaf spring where b=500 mm

Num. of node / Radius Mm	Stress in Node 36	Stress in Node 37	Stress in Node 38	Stress in Node 39	Stress in Node 40	Stress in Node 41	Stress in Node 42	Stress in Node 43	Stress in Node 44	Stress in Node 45	Stress in Node 46
1250	59	23	157	26	167	25	168	27	160	23	59
1500	59	24	155	28	169	27	170	29	158	24	57
1750	58	24	150	28	166	28	166	29	152	24	55
2000	58	25	151	30	165	29	166	30	154	25	54
2250	58	29	149	30	164	29	165	31	152	25	53
2500	56	25	144	30	159	29	160	30	146	25	52
2750	58	26	148	31	165	31	166	32	150	26	52
3000	56	26	145	30	159	30	160	31	147	25	51



Distance along x-axis for first leaf spring(1) in mm   Distance along x-axis for second leaf spring(2) in mm   Distance along x-axis for third leaf spring(3) in mm





Distance along x-axis for forth leaf spring(4) in mm
Distance along x-axis for fifth leaf spring(5) in mm

Fig(4):- indicates the stress distribution in upper and lower surface of the five leaves

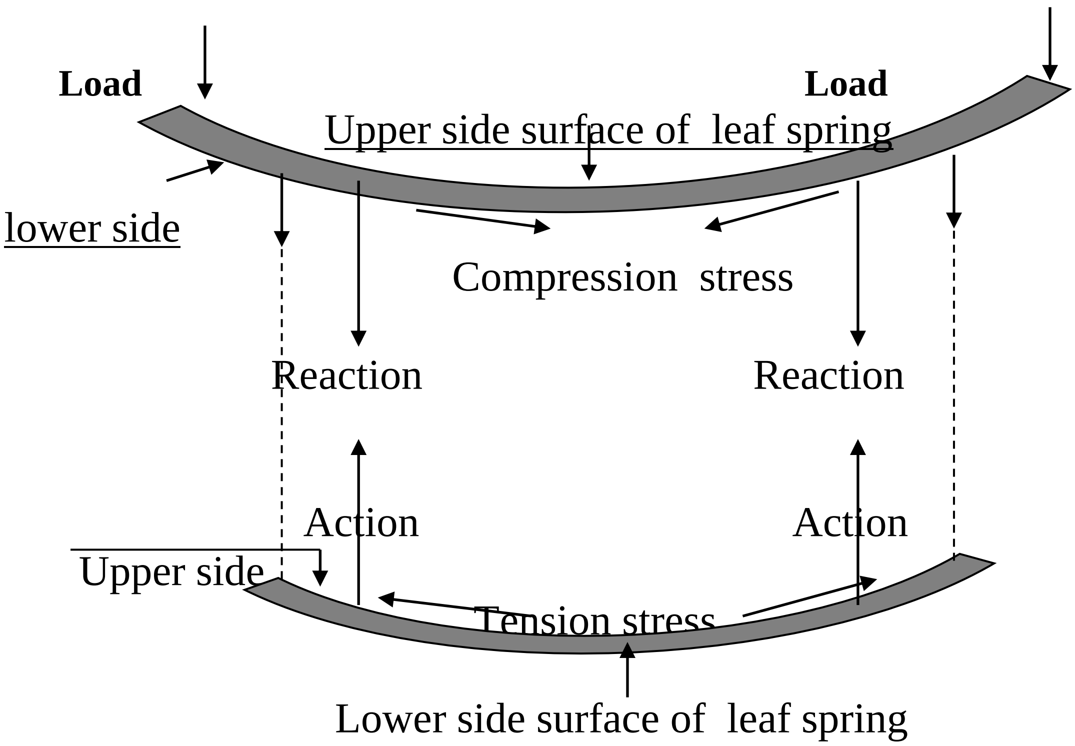


Fig. (5):- indicates (the upper and lower side), (the action and reaction forces) and (tension and compression stress)

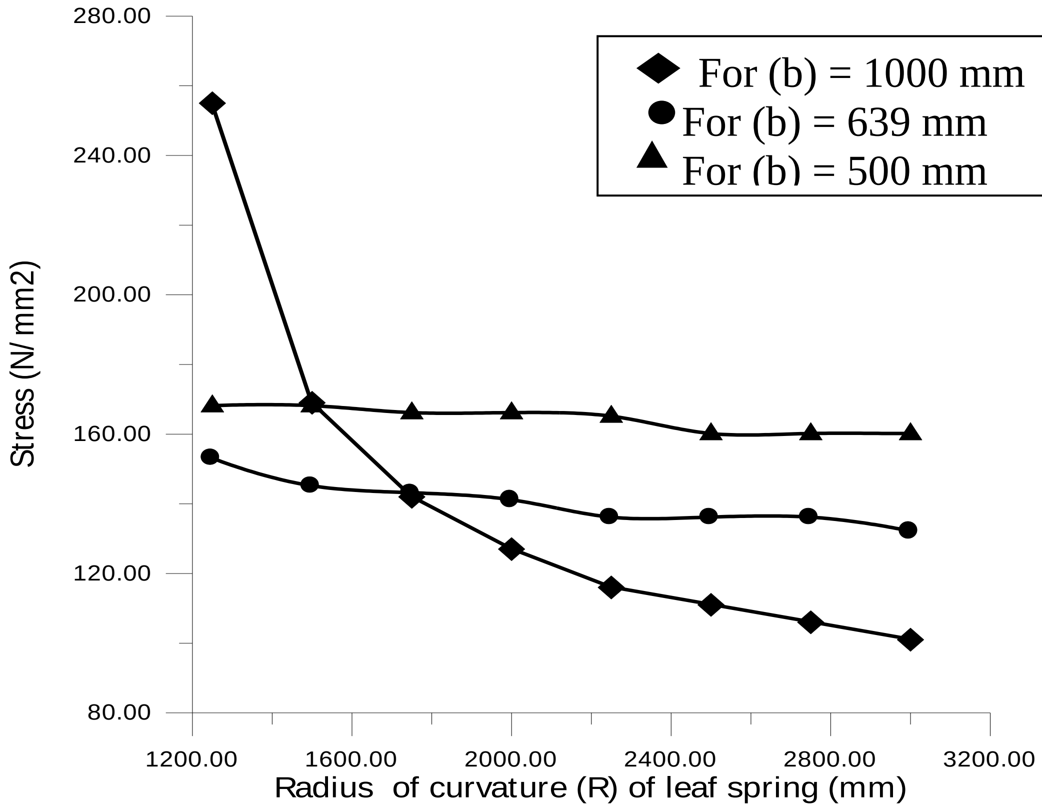


Fig. (6):- indicates the relationship between the maximum stress and radius of curvature for different magnitudes of length (b) for first leaf

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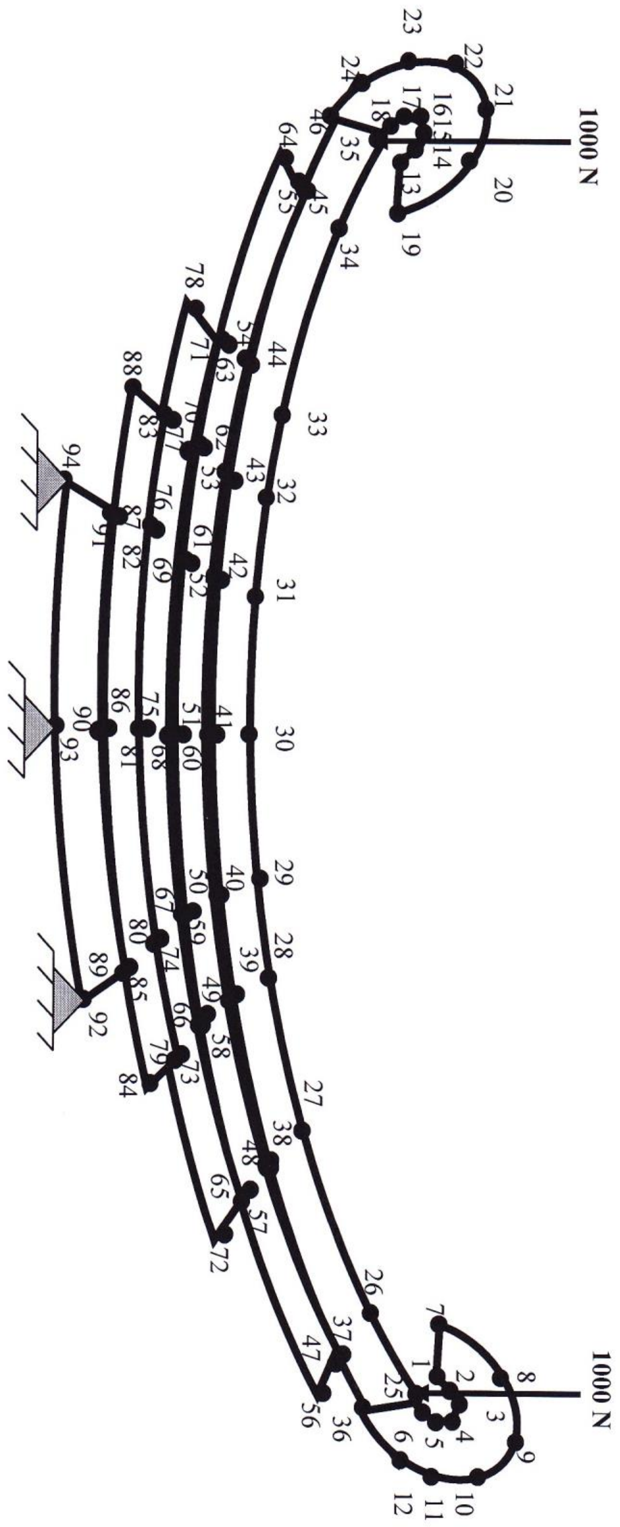
Appendix (a)



دراسة تحليل الاجهادات لنابض ورقي باستخدام طريقة العناصر المحدد

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الخلاصة

تعد الشاحنات الخفيفة من الأشياء المهمة والحاسمة إلى حد بعيد في نمو الاقتصاد العالمي منذ أن استخدم في نقل السلع والبضائع المختلفة. يعد النابض الورقي أحد الوسائل المعتبرة التي تستعمل في نظام التعليق, الخاص بأنواع معينة من الشاحنات بسبب قلة كلفة إنتاجه ودرجة الوثوقية به و سهولة صيانته. في هذه الدراسة تم استخدام تقنية معينه لتحري تصميم النابض الورقي الخاص بالشاحنات الخفيفة. لقد تم استخدام محددات معينه لتناسب وضع النابض في ألشاحنات الخفيفة لفحص توزيع الاجهادات على كل ورقه من أوراق النابض.في هذا البحث تم تحديد مواصفات النابض الورقي التابع لشاحنة خفيفة الحمل وكما يلي:

عدد أوراق النابض الورقي هي خمس أوراق سمك كل ورقة هو(10ملم) , طول المسافة من الخط الشاقولي الذي ينصف النابض الورقي إلى نصفين متماثلين ولغاية أحد طرفي النابض الورقي هو (1000~500)ملم وأنصاف أقطار التقوس للنابض الورقي هو (3000~1250)ملم أما الحمل فقد تم تسليط حمل على طرفي النابض مقداره 1000 نيوتن لكل طرف. وتكون عدد الحالات المدروسة تساوي ٢٤ حالة. إن الطريقة المستخدمة في التحليل هي طريقة العناصر المحددة لاختبار توزيع الأحمال على كل ورقة من النابض.

الكلمات الدالة:- تحليل الاجهادات النابض الورقي.