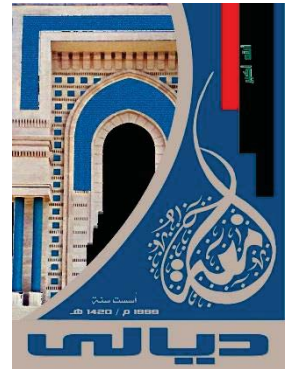




وزارة التعليم العالي والبحث العلمي  
جامعة ديالى | كلية العلوم  
قسم الرياضيات



## إيجاد القيم الذاتية لمسائل القيمة الذاتية اللاخطية باستخدام المؤثرات التفاضلية الزائفة

رسالة مقدمة الى

كلية جامعة ديالى | كلية العلوم | قسم الرياضيات وهي جزء من متطلبات نيل درجة  
الماجستير في علوم الرياضيات

من قبل

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بإشراف

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# *Chapter One*

*General Introduction*

## Chapter One: General Introduction

### 1.1 An Overview

Pseudo differential operators is developed from the theory of singular integral operators, and it gives differential and integral operators a uniform treatment it rely heavily on the Fourier transformation and its inverse. In 1960, the pseudo differential operators were first defined by Friedrichs and Lax in the study of singular integral differential operators to be able to invert differential operators to solve differential equations.

Linear pseudo differential operators are denoted by symbols, which are generalized Fourier multipliers. The class of pseudo differential constitute an algebra, and algebraic calculations of the related symbols can be used to investigate the operations of composition, transposition, and adjoining of operators, which permit us to have a facilities in the treatment of this kind of operators.

Moreover, we conclude that the class of pseudo differential operators is invariant under diffeomorphic coordinate transformations. As linear mappings between distributions. For elliptic pseudo differential operators, we construct parametrices, for elliptic operators. In addition we construct a fundamental solutions if they exist.

### 1.2 Related Works

The pseudo differential operators were used in many mathematical domains and their applications such as modern analysis and mathematical physics. They also have a particular importance to study the elliptic operators and in the subject of index theory.

In 1984, Didier Robert has used the pseudo differential operator's technique to prove the existence of the non-trivial eigenvalues for some non-linear eigenvalues problems with small parameter.

Dauge-Robert [1987], BouzouinaRobert [2002] have defined a formula of composition for Weyl-symboles of pseudo differential operators.

In 2004, Didier Robert has generalized the results proved by Helffer, Robert and Wang, 2005, and in addition he proved the existence of infinite number of eigenvalues for an even dimensional space.

In 2009, Aboud Fatima, has proved the existence of non-trivial eigenvalues for some kind of quasi-homogenous non-linear eigenvalue problems using the pseudo-differential operators techniques.

### **1.3 Problem Statement**

Difficulty finding non-trivial eigenvalues for some types of nonlinear eigenvalue problems such as cubic families of nonlinear eigenvalues problems and quasi-homogeneous families of ternary type quasi-homogeneous operators.

### **1.4 The aim of the thesis**

Study a family of semi-homogeneous operators and obtain results. Indicate the existence of non-trivial eigenvalues by making use of the facilities obtained taking advantage of the facilities obtained by the symbols of pseudo differential operators when calculating parameters.

### **1.5 Thesis Organization**

The following chapters make up the remainder of the existing work:

#### **Chapter Two: Fundamental Notions for Pseudo Differential Operators**

It contains a comprehensive overview of pseudo differential operators for example eigenvalue, Fourier transform and Schwartz space as well as introduces basic concepts that were used in the present work.

#### **Chapter Three: Cubic and Quadratic Family of Pseudo Differential Operators**

The existence of a total eigenvector system for some nonlinear eigenvalue problems has been demonstrated by applying the properties of a pseudo differential operator.

#### **Chapter Four: Quasi-homogenous family of pseudo Differential operators**

In this chapter, we study a family of quasi-homogeneous operators, and use the facilities obtained by pseudo differential operator symbols when calculating the parameter and obtain very important results indicating the existence of non-trivial eigenvalues.

#### **Chapter Five: *Conclusions and Suggestions for Future Works***

This chapter presents conclusions from the results of the presented work and some suggestions for future works.

## الخلاصة:

في رسالتنا هذه نعطي تذكير ببعض المفاهيم الأساسية المتعلقة بالمؤثر التفاضلي الزائف، كما نقدم الفرضيات المتعلقة برموز المؤثرات التفاضلية الزائفة و نقوم بتعريف الاشعة ذات التناقص الأدنى لأجل التمكن من برهنة وجود نظام تام من المتجهات الذاتية. نطبق هذه النتائج لعائلة مكعبة لمسائل القيم الذاتية اللاخطية.

بالإضافة الى ذلك، ندرس مسألة قيمة ذاتية لا خطية لعائلة من المؤثرات شبه المتجانسة، وذلك بالاستفادة من التسهيلات الناتجة عن استخدام رموز المؤثرات التفاضلية الزائفة عند حساب البارامتر كس للحصول على نتائج تؤدي الى برهنة وجود قيم ذاتية غير تافهة لمسائل قيم ذاتية لا خطية ثلاثية النوع الشبه متجانس.