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Chapter One:
Basic Definitions and Elementary Concepts

Introduction

In this chapter, presenting a review of the basic concepts and primary definitions that were used in accomplishing this work and that were used in the subsequent chapters.

1.1 Dimensions [7]

A dimension is the measure by which a physical variable is expressed quantitatively. In fluid mechanics, there are three primary dimensions from which all the dimensions can be derived which are mass (M), length (L) and time (T). All other variables in fluid mechanics can be expressed in terms of M, L and T. For example, acceleration has the dimensions LT^{-2}. Force is directly related to mass, length, and time by Newton's second law ($Force = mass \times acceleration, F = \mathrm{m} \times \mathrm{a}$). From this one can see that, the force has the dimensions MLT^{-2}.

1.2 Basic Definition and Fluid Properties

This section contains some important definitions related to fluid properties.

1.2.1 Fluid [38]

Fluid is defined as a substance that deforms continuously under the action of shear stress, regardless of its magnitude.

1.2.2 Fluid Mechanics [38]

The subject of fluid mechanics deals with the behavior of fluids when subjected to a system of forces. This subject divided into three fields:

- **Statics**: it deals with the fluid elements, which are at rest relative to each other.
- **Kinematics**: it deals with the effect of motion, i.e., translation, rotation, and deformation on the fluid elements.
- **Dynamics**: it deals with the effect of applied forces on the elements.

1.2.3 Mass Density [38]

Mass density, denoted by ρ in dimension of $\mathrm{M/L^3}$, is defined as the mass per unit volume of fluid, i.e.,

$$\rho = \frac{\mathrm{m}}{\mathrm{V}} \,, \tag{1.1}$$

where m and V represent the mass and the volume, respectively.

1.2.4 Pressure [38]

The pressure is defined as the normal compressive force per unit area which is denoted by p in dimension of $\mathrm{M/LT^2}$, i.e.

$$p = \frac{Force}{Area} = \frac{\mathrm{ma}}{\mathrm{A}} \,, \tag{1.2}$$

where a is the acceleration.

1.2.5 Shear Stress [38]

□hear stress is defined as the force per unit area, mathematically which is denoted by τ in dimension of $\mathrm{M/LT^2}$, i.e.

$$\tau = \frac{F}{\mathrm{A}} \,, \tag{1.3}$$

where τ, F, and A represented the shear stress, the applied force, and the cross□sectional area of material with an area parallel to the applied force vector respectively.

1.2.6 Shear Strain [7]

□hear strain is also known as shear deformation of a solid body is displaced parallel planes in the body□quantitatively it is the displacement of any plane relative to a second plane divided by the perpendicular distance between planes causing such formation.

1.2.7 Viscosity [11]

□iscosity is defined as the attribute the fluid which offers opposition to the motion of one coat of fluid over another neighboring coat of the fluid.

$$\mu = \frac{\tau}{du/dy} \qquad (□.□)$$

The shear stress may be defined by the proportional to the rate of change of velocity with the respect to y. it is denoted by:

$$\tau = \mu \, du/dy \qquad (1.5)$$

The du/dy represented the rate shear strain or rate of deformation or velocity gradient. As one we can defined the viscosity as the shear stress required to produce unit.

1.2.8 Dynamic Viscosity [11]

□ynamic viscosity, denoted by μ in dimension of M/LT, is defined as the tangential force required per unit area to sustain a unit velocity gradient, i.e.

$$\mu = \frac{F/A}{du/dy} \, . \qquad (1.6)$$

1.2.9 Kinematic Viscosity [11]

□inematic viscosity of the fluid, denoted by υ in dimension of L^2/T, is defined as the ratio of dynamic viscosity to density of the fluid and it is related with the dynamic viscosity through the equation as follow:

$$\upsilon = \frac{\mu}{\rho} \, , \qquad (1.7)$$

where μ and ρ represents the dynamic viscosity and the density, respectively.

1.2.10 Stream Function [11]

It is defined as a scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component a right angles to that direction. It is denoted mathematically by ψ, in unit of L^2/T, where $\psi = \psi(x, y, t)$. For two dimensional unsteady flows, we have:

$$u = \frac{\partial \psi}{\partial y} \;, \quad v = -\frac{\partial \psi}{\partial x} \;, \tag{1.8}$$

where u and v are the velocity components in the x and y direction, respectively.

1.2.11 Streamline [11]

A streamline is an imaginary line drawn through the flow field, such that the tangent at any point is in the direction of the velocity vector.

1.3 Newton's Law of Viscosity [15]

The Newton's law of viscosity states that the shear stress τ of fluid element on a layer is directly proportional to the rate of strain $\tau \propto (du/dy)$ which may be written as:

$$\tau = \mu \frac{du}{dy} \;, \tag{1.9}$$

in which μ is a constant of proportionality and is known as coefficient of dynamic viscosity or simply viscosity. Further, $\frac{du}{dy}$ represent the rate of shear strain (velocity gradiant), a normalized measure of deformation representing the displacement between particles in the body relative to a reference length

1.4 Types of Fluid Flow [38]:

A fluid flow consists of a flow of the number of small particles grouped. These particles may group themselves in a variety of ways and type of flow depends on how these groups behave. The following are the important types of fluid flow:

❖ **Steady and Unsteady Flow:** A flow is considered to be steady when conditions at any point in the fluid do not change with time, i.e., $\frac{\partial u}{\partial t} = 0$ and also the properties do not change with time, i.e., $\frac{\partial p}{\partial t} = 0, \frac{\partial \rho}{\partial t} = 0.$

A flow is unsteady when condition or conditions at any point change with time, i.e., $\frac{\partial}{\partial t}(.) \neq 0.$

❖ **Laminar and Turbulent Flow:** Laminar flow in which fluid particles move along smooth paths in laminar or layers, with one layer gliding smoothly over an adjacent layer and it occurs for the value of Renold's number from 0 to 2000. We say that the flow is turbulent if the fluid particles move in very irregular parts and when Renold's number is greater than □□□□, and we say that the flow is translation if the values of Reynold's number between 2000 and □□□□.

❖ **Compressible and Incompressible Flow:** A flow is considered to be compressible if the mass density of fluid ρ changes from point to point, or $\rho \neq$constant. In the case of incompressible flow, the change of mass density in the fluid is neglected or density is assumed to be constant.

1.5 Classification of Fluids [7][15].

The fluids may be classified into the following types:

❖ **Ideal Fluid:** A fluid, which is incapable of exerting shearing stress whether at rest or in motion or it is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluid, which exists, has some viscosity.

- **Real Fluid:** A fluid, which poses viscosity or exerting shear stress, is known as real fluid. All the fluids in actual practice are real fluids.
- **Newtonian Fluid:** A real fluid in which shear stress is directly proportional to the rate of shear strain (Linear relation), is known as Newtonian fluid, i.e. obeys the Newtonian's law of viscosity, for example air, water, and gasoline
- **Non-Newtonian Fluid:** A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), is known as Non-Newtonian fluid. For example honey, blood, processing of food are considered to be non-Newtonian fluid, etc.
- **Ideal Plastic Fluid:** A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradiant), is known as ideal plastic fluid.

1.6 Continuity Equation [29]

The continuity equation simply expresses the law of conservation of mass (the mass per unit time entering the tube must flow out at the same rate), which has the following mathematical form:

$$\frac{D\rho}{Dt} + \rho\nabla.\vec{\boldsymbol{V}} = 0\,, \tag{1.11}$$

where ρ is density and $\vec{\boldsymbol{V}} = (u, v, w)$ is the velocity components in (x, y, z) directions, $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ is gradient, and $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{\boldsymbol{V}}.\nabla$ is the substantial derivative.

The continuity equation may be expressed in the three most commonly used coordinate systems, namely rectangular, cylindrical, and spherical. In our work we used only rectangular (Cartesian) systems which defined as follows:

1.6.1 Continuity Equation in Cartesian Coordinates [29]

The equation of continuity in three□dimensions is:

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0\,. \tag{1.12}$$

If the fluid is incompressible, $\rho=$ constant, and the continuity equation may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0\,. \tag{1.13}$$

1.7 Momentum Equations [46]

The system of nonlinear partial differential equations that describe the fluid motion is called the momentum equations or □avier□□tokes equations. The general technique for obtaining the equations governing fluid motion is to consider a small control volume through which the fluid moves and required that mass and energy are conserved, and that the rate of change of the two components of linear momentum is equal to the corresponding components of the applied force.

1.7.1 The Momentum Equations in Cartesian Coordinates

The momentum equations (□avier□□tokes equations) in cartesian coordinates are:

$$\left.\begin{aligned}
\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}&=F_x-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\nabla^2 u\\
\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}&=F_y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\nabla^2 v\\
\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}&=F_z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\upsilon\nabla^2 w
\end{aligned}\right\} \tag{1.14}$$

where (u, v, w) are the velocity components in the $x, y,$ and z directions respectively, (F_x, F_y, F_z) are the body forces in the $x, y,$ and z directions respectively, ρ is the mass density, p is the pressure and υ is the kinematic viscosity, and $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is a Laplacian operator.

1.8 Peristaltic Phenomenon [9]

□eristaltic transport is a unique mechanism of fluid transport by way of area contraction□expansion of progressive waves. Also, it is an inherent property of many of the physiological systems, such as blood flow in small vessels of the human circulator system, transport of the ovum and cilia, urine transport from the kidney to the bladder through ureters, bile flow from the gallbladder into the duodenum, transport of food in the digestive tract. □eristaltic motion in the industrial applications is employed in the transport of corrosive and noxious fluids, hose pumps, tube pumps, dialysis machines, and heart□lung machines.

□everal theoretical and experimental attempts have been made to examine the peristaltic flows under various assumptions of long wavelength, low □eynolds number, small wave number, small amplitude ratio, etc.

1.9 Porous Medium [39]

A porous medium is a material containing pores (voids). The pores are typically filled with a fluid (liquid or gas). Many natural substances such as rocks and soil, biological tissues (e.g. bones, wood, cork), and manmade materials such as cements and ceramics can be considered as porous media. Many of their important properties can only be rationali□ed by considering them to be porous media. The concept of porous media is used in many areas of applied science and engineering: filtration, mechanics (acoustics, soil mechanics, rock mechanics), engineering (petroleum engineering, bio□remediation, construction engineering), geosciences (hydrogeology, petroleum geology, geophysics), biology and biophysics, material science, etc.

The □avier –□tokes equations with porous medium are given by:

$$\rho \frac{D\vec{\boldsymbol{V}}}{Dt} = \rho \vec{F} - \nabla \vec{p} + \upsilon \, \nabla^2 \vec{\boldsymbol{V}} - \frac{\mu}{\bar{k}} \vec{\boldsymbol{V}} \,, \tag{1.15}$$

where $\bar{k}$ is the permeability parameter.

1.10 Basic Definitions on the Electrostatic Field [24]

Through this section, the most important definitions, which will be used in this work later on, are introduced.

1.10.1 Electrostatic Field

The electrostatic field $\vec{\boldsymbol{E}}$ is defined as the force that is exerted on a unit charge, in the field and its vector in the same direction as the force.

$$\vec{\boldsymbol{E}} = \lim_{\Delta q \to 0} \frac{\vec{\boldsymbol{F}}}{\Delta q}\,, \tag{1.16}$$

where Δq is the charge and $\vec{\boldsymbol{F}}$ is the force.

1.10.2 Current Density

The definition of the current density, denoted by $\vec{\boldsymbol{J}}$ as the flow of charges across a unit cross sectional area per second.

1.10.3 Electrical Conductivity

It is denoted by σ' and is defined by the ratio of current density $\vec{\boldsymbol{J}}$ to electrostatic field $\vec{\boldsymbol{E}}$.

$$\sigma' = \frac{\vec{\boldsymbol{J}}}{\vec{\boldsymbol{E}}} \tag{1.17}$$

1.10.4 Ohm's Law [10]

The Ohm's law describes the conduction current, and is given by:

$$\vec{\boldsymbol{J}} = \sigma'\left(\vec{\boldsymbol{E}} + \vec{\boldsymbol{V}} \times \vec{\boldsymbol{B}}\right), \tag{1.18}$$

where σ' is electrical conductivity, $\vec{\boldsymbol{E}}$ is the electrostatic field, $\vec{\boldsymbol{V}}$ is velocity and $\vec{\boldsymbol{B}}$ is magnetic field.

1.10.5 Lorentz Force

The Lorent□force, denoted by $\vec{\boldsymbol{F}}_p$, on a charge moving in a magnetic field with velocity $\vec{\boldsymbol{V}}_p$ is given by:

$$\vec{\boldsymbol{F}}_p = q\vec{\boldsymbol{V}}_p \times \vec{\boldsymbol{B}}\,, \tag{1.19}$$

and if $\vec{\boldsymbol{J}} = q\vec{\boldsymbol{V}}_p$ then

$$\vec{\boldsymbol{F}}_p = \vec{\boldsymbol{J}} \times \vec{\boldsymbol{B}}\,. \tag{1.20}$$

1.11 Magnetohydrodynamics (MHD)[19]

Magnetohydrodynamics (M□□) is a branch of continuum mechanics, which deals with the motion of electrically conducting fluid in the presence of a magnetic field. The motion of conducting material across the magnetic lines of force creates potential differences which, in general, cause electric currents to flow. The magnetic fields associated with these currents modify the magnetic field which creates them. In other words, the fluid flow alters the electromagnetic state of the system. □n the other hand, the flow of electric current across a magnetic field is associated with a body force, the so□called Lorent□force, which influences the fluid flow. It is this intimate interdependence of hydrodynamics and electrodynamics which really defines and characteri□es M□□.

The M□□ □avier□□tokes equations are presented as follows:

x −direction

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho F_x - \frac{\partial p}{\partial x} + \mu\nabla^2 u + \left(\vec{J} \times \vec{B}\right)_x, \tag{1.21}$$

y −direction

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho F_y - \frac{\partial p}{\partial y} + \mu\nabla^2 v + \left(\vec{J} \times \vec{B}\right)_y, \tag{1.22}$$

z −direction

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho F_z-\frac{\partial p}{\partial z}+\mu\nabla^2 w+\left(\vec{J}\times\vec{B}\right)_z, \tag{1.23}$$

in which $\left(\vec{\boldsymbol{J}}\times\vec{\boldsymbol{B}}\right)_x, \vec{\boldsymbol{B}}, \vec{\boldsymbol{J}}, \mu$ are the component of Lorent□force (electromagnetic force) in the x −direction, the magnetic field, the current density or conduction current, and dynamic viscosity respectively.

1.12 Heat Transfer [44]

□henever a temperature difference exists in a medium or between media, heat transfer must occur. □eat transfer is energy in transit due to a temperature difference. Three different types of heat transfer processes are known. □hen a temperature gradient exists in a stationary medium, which may be a solid or a fluid is used the term conduction for the heat transfer that occurs across the medium. The term convection refers to heat transfer that occurs between a surface and a moving fluid when they are at different temperatures. The third kind of heat transfer is termed thermal radiation. All surfaces of finite temperature emit energy in the form of electromagnetic waves. □wing to radiation between two surfaces at different temperatures when there is an absence of an intervening medium a net heat transfer occurs.

1.12.1 Heat Flux

The heat flux Q is defined by the Fourier□ law as the rate of heat flow is proportional to the area of flow A and the temperature difference (dT), across the layer, and inversely proportional to the thickness dx, and varies only in one direction, say x. It is expressed mathematically by

$$Q=-\kappa A\frac{dT}{dx}, \tag{1.24}$$

where k is the thermal conductivity, A is the area and the negative sign indicates that the temperature change in the direction of heat flow $(-dT)$.

1.12.2 Specific Heat of Fluid at Constant Pressure

It is defined as the ratio of heat flow to mass and temperature difference and denoted by C_P which is given as

$$C_P = \frac{dQ^*}{dT}\,, \tag{1.25}$$

1.12.3 Thermal Conductivity

It is denoted by (κ) which is defined as the flow in a unit time across unit area through unit thickness when a temperature difference of unity is maintained between opposite faces.

1.12.4 Heat Capacity [13]

It is denoted by (ρc_p), which is defined by the product of density and specific heat.

1.13 The Energy Equation[19]

The energy equation is represented by a non-linear partial differential equation and is given by

$$\rho c_P \frac{DT}{Dt} = \Phi + \kappa \nabla^2 T + \rho R\,, \tag{1.26}$$

where $\frac{DT}{Dt} = \left(\frac{\partial}{\partial t} + \vec{\boldsymbol{V}} \cdot \nabla\right) T$ and R is the rate at which heat is added by chemical reaction, radiation, and electromagnetic action, Φ is called the dissipation function and can be shown that Φ the rate at which work is converted into heat is greater or equal to zero which equal to

$$\Phi = \lambda\left(\nabla \cdot \vec{\boldsymbol{V}}\right)^2 + 2\mu D \cdot D\,, \tag{1.27}$$

المستخلص

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