تحديد الحرارة على حدود مسألة عكسية لمعادلة من نوع هولموتز

رسالة مقدمة إلى مجلس كلية العلوم- جامعة ديالى
وهي جزء من متطلبات نيل درجة الماجستير في علوم الرياضيات

من قبل الطالبة

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Chapter One

Some basic concepts

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1.1 Introduction

In this chapter, we recall some basic concepts that we need in our work

1.2 Some Basic definitions and properties

Definition1.2.1(Well posed and ill-posed problems)[10]: A well posed problem is defined by satisfy the following properties:

1. The problem must have a solution (existence)

2. The solution of the problem must be unique (uniqueness), and

3. The problem must depend continuously on the given data (stability)

In case that any of these three conditions is missing, then the problem is said to be ill-posed.

Definition1.2.2(Direct problems)[25]: Adirect problem is a well-posed problem with the uniqueness, stability and existence of a solution of the corresponding mathematical problem.

Definition1.2.3(Inverse problems)[25]: In case that one of the conditions needed to solve the direct problem is missed then we call the problem an inverse problem. Where inverse problems appear in places that are difficult to reach by direct methods, such as extremely hot places and the process of extracting oil from the ground, and it was introduced in the field of medicine, for example, a problem in the heart of a patient or in the brain of a patient, which cannot be reached by direct methods. Here we use the inverse problem.

Definition1.2.4(Boundary conditions)[27]: Boundary conditions are necessary constraints for the solution of a boundary value problem. A boundary value problem is a differential equation (or system of differential equations) to be solved in a domain on whose boundary a set of conditions are known.

1. Dirichlet boundary conditions.
2. Neumann boundary conditions.
3. Mixed boundary condition**.**
4. Robin boundary condition. The Robin boundary condition is a type of boundary condition which consists of a linear combination of the values of the field and its derivatives on the boundary.
5. Specifying a weighted average of first and second kinds is called Cauchy boundary conditions.

Definition1.2.5(Inverse Cauchy problems)[1]: A problem is called a inverse Cauchy problem in case that the boundary conditions for both the solution and its normal derivative are prescribed only on a part of the boundary of the solution domain, but no information is available on the other part of the boundary.For example Inverse Cauchy problems for the Helmholtz equation.For example (Inverse Cauchy problems for the Helmholtz equation, inverse Cauchy problem for the Poisson equation, …   )

Definition1.2.6(Instability)[25]: The instability of the solution means that a small measurement error in the input data may amplify significantly the errors in the solution.

Definition1.2.7 (Smooth boundary)[33]: A smooth boundary is the boundary of a domain for which we can find a function which is continuous with all its derivatives on this boundary and its closure.

Definition1.2.8(Helmholtz equation)[2]: The Helmholtz equation is given by

$$\Delta T + k^2 T = F,$$

$$\text{with some boundary conditions}$$

$$T(\rho, \theta) = \tilde{T}(\theta), 0 \leq \theta \leq \beta\pi$$

$$\partial_n T(\rho, \theta) \equiv \Phi(\rho, \theta) = \tilde{\Phi}(\theta), 0 \leq \theta \leq \beta\pi$$

Where k is a physical parameter.

Definition 1.2.9(Laplacian operator)[9]: The Laplacian operator is defined as follows.

$$\Delta T = \nabla . \nabla T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$

The gradient of T is define by

$$grad\, T \equiv \nabla T = \frac{\partial T}{\partial x}\vec{\imath} + \frac{\partial T}{\partial y}\vec{\jmath}$$

Definition1.2.10(Normal derivative)[31]: A direction derivative which is outward normal (orthogonal) to some curve, surface or hypersurface is called the normal derivative and it is done by the inner product of the gradiant with the normal vector.

$$\frac{\partial \mathrm{T}}{\partial \mathrm{n}} = \nabla \mathrm{T}.\vec{\mathrm{n}}$$

Definition 1.2.11 (Inner product)[31]: The product matrix of size $1 \times n$ (the row vector x^T) with an $n \times 1$ matrix (the column vector y) . Let $x = (x_1, \cdot \cdot \cdot, x_n), y = (y_1, \cdot \cdot \cdot, y_n)$

The inner product is defined by

$$x^T.y = [x_1 \cdot \cdot \cdot x_n]\begin{pmatrix} y_1 \\ \vdots \\ \vdots \\ \vdots \\ y_n \end{pmatrix} = x_1 y_1 + \cdots + x_n y_n$$

Definition 1.2.12(A norm of a vector)[31]: A norm of a vector x, denoted by $\|x\|$, is a real function with the properties

- $\|x\| \geq o$
- $\|x\| = o \quad if\ and\ only\ if\ x = 0$
- $\|x + y\| \leq \|x\| + \|y\|$
- $\|\alpha x\| = |\alpha| \|x\|$

Where x and y are vectors and for a scalar α

Definition1.2.13 (Symmetric and positive definite matrices) [31]: A square matrix A of order n is called symmetric if $A = A^T$, that is, the matrix equals its transpose, the so called positive definite matrices. A square matrix A is positive definite if

$$x^T A\, x > 0, \quad x \neq 0$$

Definition1.2.14 (Meshless Methods)[22]: The meshless method is a method unlike the mesh-based method (finite difference or volume , finite element , and boundary element,…). The meshless method using node points within a problem domain without the need for nodal connectivity.

Definition1.2.15(Errors)[31]: In general, there are two methods of measuring the errors, the first is the absolute error and the second is the relative error. Which is the absolute error is given by the absolute value of the difference between the approximate (calculated) solution and the exact (true) solution. The relative error is defined by the division of the absolute error by the exact solution.

Definition1.2.16(Tolerance)(Tol) [30]: Where Tol is a given value for error tolerance chosen by the programmer, this value habitually chosen to be near to zero.

Definition1.2.17(Preconditioning)[31]: The preconditioning is a numerical method used for the ill-conditioned problem, by multiplying it by a matrix called preconditioner which has some appropriate advantage to reduce the condition number of the matrix corresponding the problem.

Definition1.2.18(Condition number)[31]: The condition number of an invertible matrix A is defined as

$$\kappa(A) = \|A^{-1}\| \, \|A\|$$

recalling that problem is called **well-conditioned** if it has a small condition number and it is called **ill-conditioned** if it has a big condition number.

المستخلص

هذه الرسالة تهدف الى حل مسائل كوشي العكسية عدديا والمعرفة على مجال ثنائي البعد مشغول بمادة تحقق معادلة من نوع هولموتز و ذلك بالأستفادة من الشروط الكوشية الإضافية المعرفة على الجزء الحدودي الذي يمكن الوصول إليه. تم استخدام طريقة عددية لا شبكية بأستخدام تقريب متعدد الحدود للحل. لتأكيد كفاءة الطريقة المقترحة، تم دراسة أمثلة مختلفة وتم حل النظام الخطي الناتج بأستخدام خوارزميتين CGM و CGLS. تم التحقق من استقرارية الطريقة المقترحة بتطبيق عوامل ضوضاء مختلفة. ولتجنب خاصية الشرط الشديد السوء لمسائل الكوشية العكسية ، تم تطبيق التنظيم والتكييف المسبق للحصول على دقة تقريب أفضل لكلا الخوارزميتين CGM و CGLS.

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