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رسالة

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Chapter One

General

Introduction

1.1 Introduction

A fundamental idea in mathematics is a crisp set. A crisp set is one that consists of elements that have either full or no membership in the set. In set theory, the principle notion is defined as an element assigned a value of 1 or 0 to reflect whether it belongs or does not belong to the set.

Fuzzification may be defined as the process of transforming a crisp set to a fuzzy set or a fuzzy set to fuzzier set. Basically, this operation translates accurate crisp input values into linguistic variables. In a number of engineering applications, it is necessary to defuzzify the result or rather "fuzzy result" so that it must be converted to crisp result. Fuzzification is done by recognizing various assumed crisp quantities as the non-deterministic and completely uncertain in nature. This uncertainty may be emerged because of imprecision and uncertain that lead variables to be presented by a membership function because they can be fuzzy in nature. Fuzzification translates the crisp input data into linguistic variables which are represented by fuzzy sets. After that, it applied the membership functions to measure and determine the degree of membership [1].

Fuzzy calculus has emerged as an important area of investigation because of its widespread applications in various fields like medical science, physics, chemistry, bio-mathematics, and groundwater problems. Many developments have taken place in fuzzy calculus in the last few decades. The concept of fuzzy calculus originates from the idea of the extension of classical derivative and integration to fuzzy derivative and integration. Precisely, fuzzy calculus is the study of theory and applications of integrals and derivatives of uncertain functions. This branch of mathematical analysis, extensively investigated in recent years, has emerged as an effective and powerful tool for the mathematical modeling of several engineering and scientific phenomena. The uncertainty is an important concept and its understanding is necessary for decision making. Uncertainty means that you have the information about phenomenon of nature less than the total information required to this phenomenon and its environment [2].

The purpose of study fuzzy theory is to describe situations in which the data are imprecise or vague. The fuzziness phenomenon more accurate in appropriateness and more quantified in qualitative analysis appropriateness so as to realize the more objective and accurate effect than pure usage of qualitative analysis. The proposed techniques for solving our fuzzy problem has been successfully employed to produce exact and precise approximate solutions by involving fast convergent power series for emerging realism models in physical phenomena due to its features, which are that it is easy, straightforward, handles directly to various kinds of initial conditions, needs no to linearization or restrictive assumptions, does not need major computational requirements and is performed with less time and more accuracy.

1.2 Historical Background

Zadeh addressed this topic in 1965 essay, " As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics " [3].

Although the tools of fuzzy calculus have been available and applicable to various fields of study, the investigation of the theory of Fuzzy Differential Equations (FDEs) has only been started quite recently.

Zadeh, (1965) [4] used the membership function to propose the concept of fuzzy sets for the first time.

The (FDEs) was first introduced by Kandel et.al (1978) [5]. The numerical methods for solving (FDEs) in the following form are introduced by Abbasbandy et.al (2002) [6].

$$y'(t,r) = g\big(t, y(t,r)\big) \tag{1.1}$$

$$y(t_0, r) = y_0(r)$$

Fuzzy derivative and its generalizations were introduced by Seikkala [7]. In a series of theoretical and practical studies such as (Friedman, & Kandel (1999) [8], Abbasbandy & Viranloo (2002) [9], Chalco-Cano & Roman-Flores (2008) [10], Ahmad et al. (2013) [11],) various formulas to determine the solutions of (FIDEs) were explored.

Many authors studied the existence and uniqueness of solutions for (FDEs) under various kinds of conditions and obtained many meaningful results. In (1987) [12] studied differentiability and integrability properties of such functions and give an existence and uniqueness theorem for a solution to a (FDEs).

In (1992) [13] discussed the modelling utility of fuzzy differential inclusions associated with given systems of nonlinear (FDEs).

In (1999) [8] considered numerical algorithms for solving Fuzzy Ordinary Differential Equations. Also, they discussed a scheme based on the classical Euler method in detail, and this was followed by a complete error analysis, the algorithm was illustrated by solving several linear and nonlinear fuzzy Cauchy problems.

In (2000) [14] have given a very general formulation of a fuzzy first-order initial value problem. They first find the crisp solution, fuzzify it and then check to see if it satisfies the (FDEs).

In (2005) [15] proved the existence and uniqueness of the solution for fuzzy stochastic differential equation under suitable Lipschitz condition.

In (2009) [16] investigated the conditions of existence and uniqueness of Fuzzy Fractional Differential Equations. The Fuzzy Fractional Integro Differential Equations is the most fascinating field. They are useful for understanding phenomena that have an underlying effect. Also in (2011) [17] used variation of constant formula for first order fuzzy differential equations.

In numerical analysis, one of the earlier contributions by Ma, M., Friedman, M., & Kandel (1999) [8],the authors proposed a Fuzzy Differential Equations by to parametric ordinary differential equations and then solved by fuzzy version of Euler's method to approximate the solution of Fuzzy Differential Equations.

In (1998) [18] demonstrated the fuzzy spline wavelets to determine the solution of differential equations. Also, their advantage in using fuzzy spline wavelets for the solution of such problems was that the solution would enjoy the excellent numerical and computational characteristics of the fast wavelet transform, while retaining the explanatory power of fuzzy system.

The B-spline function is a significant role of numerical analysis and approximate solution as used of cubic B-spline in approximating solutions of boundary value problems by Munguia, and Bhatta, (2015) [19] . Also, in (2019) [20] presented approximate solutions to linear and nonlinear ordinary differential equations using Bernstein polynomials.

In (2020) [21] introduced a new scheme based on the exponential spline function for solving linear second order Fredholm integro-Differential Equations.

In (2018) [22] applied the generalized spline technique and Caputo differential derivative to solve second kind of Fractional Integro-Differential Equations.

In (2021) [23] introduced a new class of cubic spline function approach to solve fuzzy initial value problems efficiently. Also, the convergence of this method was shown. They compared of the applied method with exact solutions reveals that the method was tremendously effective.

1.3 Motivation of Thesis

The objectives of studying fuzzy set theory is to develop methods for formulating and solve examples that are highly complex or that have an inaccurate definition to be acceptable when dealt with by the familiar numerical methods. Therefore it can be considered fuzzies.

It is a kind of inaccuracy that we face when finding the mathematical formulation of a practical problem in which it is some kind of ambiguity.

1.4 Aims of Thesis

1. We introduce the class of Fuzzy Integro- Differential Equations (FIDEs) and determine the numerical formula to solve it.
2. We drive and identify the best approximate expression to find the solutions of proposed problem.
3. We extend Euler method by adding control parameters to get on best solutions.

4. Existence and uniqueness of Fuzzy Integro-Differential Equations (FIDEs) are discussed and we formulate and prove sufficient conditions for this.
5. We use the exponential spline method for solving proposed class of Fuzzy Integro- Differential Equations (FIDEs).
6. We give and discuss real data examples to illustrate the proposed numerical method to solve the models under consideration.
7. An efficient method for computing the approximate solutions for the equations under consideration is investigated.
8. We discover and develop a important relationship between exponential spline method in numerical analysis and fuzzy theory for a class of continuous fuzzy model.

1.5 The Organization

This thesis divided in to Four Chapters. *Chapter Two*, contains basic concepts and definitions about fuzzy theory and used mathematical tools. In *Chapter Three* we introduce the Numerical Approaches for Solving Fuzzy Integro-Differential Equations, and we discuss the existence and uniqueness of solutions. In *Chapter Four* ,illustrative examples are given to demonstrate the high precision and good performance of the new classes of our work. Finally, some conclusions and future works have been given in *Chapter Five*.

المستخلص

في هذه الرسالة قمنا بدراسة المعادلات التكاملية التفاضلية من النوع الثاني ، كما تمت مناقشة وجود ووحدانية هذه المعادلات باستخدام نظرية (Banach Fixed Point) وبعض الشروط الكافية وبعض الطرق العددية لحلها.

يشمل عملنا طريقتين عدديتين لحل المعادلات التكاملية التفاضلية الضبابية. الطريقة الأولى هي توسيع طريقة أويلر باستخدام معاملات التحكم ، أما الطريقة الثانية ، فتحتوي على توسيع طريقة الدالة التكعيبية (Exponential Spline) المستخدمة لهذا الغرض. تم تطبيق الطريقتين لإيجاد أفضل الحلول للمعادلات المقترحة.

أخيرًا ، تم إثبات فعالية طرقنا المقترحة من خلال النتائج المبنية على الخطأ المطلق(Absolute Error) ، أعطت الطرق المقترحة نتائج عالية الأداء والدقة. وتم تمثيل النتائج بجداول واشكال رسومية وباستخدام لغة ماتلابR2020a .