جمهورية العراق
وزارة التعليم العالي والبحث
العلمي
جامعة ديالى
كلية العلوم
قسم الرياضيات

الطرق العددية لحل مسالة كوشي العكسية لمعادلة لا بلاس الثنائية

رسالة

مقدمه الى مجلس كلية العلوم جامعة ديالى في استيفاء جزئي لمتطلبات درجة ماجستير العلوم في الرياضيات

من قبل

لقاء قاسم محمد

بإشراف

أ.م. د. فاطمة محمد عبود

1445هـ
2023م

Chapter One

Theoretical Background and Basic Concepts

Chapter One

Theoretical background and basic concepts

In this chapter, we present some basic concepts that we need in our work.

Definition 1.1 (Well- posed problems) [4]: If the following conditions are met, the problem is deemed well-posed in the Hadamard sense.

1- the presence of a solution.

2- solution singularity.

3- solution stability.

Definition 1.2 (Ill-posed problem) [4]: An ill-posed problem is one that does not satisfy one of the three Hadamard criteria.

Definition 1.3 (Direct problem) [23]: If the following criteria are met for mathematically expressing physical systems.

1-When the controlling partial differential equation is well known, the system's response is well known.

2-The geometry of the domain is well defined.

3-The full boundary as well as the starting point are clearly defined.

Because all of these factors are well understood, the situation is referred to as a direct problem.

Definition 1.4 (Inverse problem) [23]: If one of the requirements required to solve the direct problem is not met, we refer to the problem as an inverted problem.

In reality, because inverse problems are ill-posed, they are more difficult to solve than direct ones. The inverse problem is generally unstable, which makes it more difficult to consider.

In a variety of scientific domains, inverse problems have recently attracted significant attention.

Definition 1.5 (Inverse Cauchy problem) [23]: An inverse Cauchy problem occurs when certain areas of the border contain more data while others have missing or poorly defined data. We overcome the missing data from the extra data in the other border to solve this challenge.

Definition 1.6 (Laplace's equation) [6]: Laplace's equation is a form of equation first proposed by Pierre-Simon Laplace. It is a physics and mathematics phrase defined by the second order partial differential equation.

$\nabla^2 f = 0$ or $\Delta f = 0$ where $\Delta = \nabla.\nabla = \nabla^2$

"Δ " is the Laplace operator , and $\nabla.\nabla$ is the divergence operator (also denoted by "div") and ∇ is the gradient operator (also symbolized "grad"), for example in case of two dimensional space the Laplacian operator defined by

$$\Delta = \boldsymbol{\nabla}.\boldsymbol{\nabla} = \boldsymbol{\nabla}^{2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Where ∇ is the gradient operator $(\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$.

If the right-hand side of Laplace's equation equals a non-zero function, it is known as Poisson's equation.

Definition 1.7 (meshless method) [24]: Meshless methods, as opposed to mesh-based approaches that use discretization techniques such as the finite difference method, finite element method, and finite volume method, describe the domain using a collection of nodes.

In numerical calculations, the approximation of the exact result creates some mistake. In the following paragraphs, we'll go through one type of error that we use in our computations.

Definition 1.8 (Absolute Error) [29]: The absolute error is the absolute value of the difference between the exact and approximate solution.

Definition 1.9 (Relative error) [29]: The relative error is the difference between the exact and approximate solutions in absolute terms.

Definition 1.10 (Condition Number) [30]: The condition number of a matrix A, indicated by $k(A)$, is defined as follows

$$\boldsymbol{\kappa}(\boldsymbol{A}) = \|\boldsymbol{A}\|\left\|\boldsymbol{A}^{-1}\right\|$$

$$\boldsymbol{\kappa}(\boldsymbol{A}) \geq \mathbf{1} \textbf{ for all A}$$

Remark: A matrix is said to be well-conditioned if it is sufficiently small and stable (little changes in the data cause modest changes in the solution).

If the condition number of a matrix is large, it indicates that the matrix is unstable (little changes in the data create large changes in the solution).

Definition 1.11 (symmetric Matrix) [28]: A rectangular matrix A whose transpose is the same as its own i.e. **A=$\boldsymbol{A^T}$.**

Definition 1.12 (Inner product) [28]: Let $\boldsymbol{x} = [\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n}]$, $\boldsymbol{y} =$.$[\boldsymbol{y_1}, \boldsymbol{y_2}, \dots, \boldsymbol{y_n}]$**,** This is how the inner product of these two vectors is described

$$< \boldsymbol{x}, \boldsymbol{y} >= \boldsymbol{x}.\boldsymbol{y}^{\boldsymbol{T}} = [\boldsymbol{x_1} \dots \boldsymbol{x_n}]\begin{bmatrix} \boldsymbol{y_1} \\ . \\ . \\ \boldsymbol{y_n} \end{bmatrix} = \boldsymbol{x_1}\boldsymbol{y_1} + \cdots + \boldsymbol{x_n}\boldsymbol{y_n}$$

Definition 1.13 (Some Types of Boundary Conditions) [24]: The following are some examples of boundary conditions:

1. Dirichlet-type Boundary Condition: If u(x) is supplied at every location, this type of boundary condition is named after the German mathematician Dirichlet. $x \in \partial\Omega$

2. Neumann –type Boundary Condition: This sort of condition is named after the German mathematician Neumann, and it occurs when the value of the gradient of the dependent variable on the boundary is referred to as the normal derivative. $\frac{\partial u}{\partial n}$ it is given at all point $x \in \partial\Omega$.

There are further boundary conditions, such as Robin, Mixed, and Cauchy, however these are beyond the scope of our investigation.

Definition 1.14 (Vector norm) [28] On R^n is a function $\|.\|$, from R^n into R with the following properties:

- $\|x\| \geq 0$ for all $x \in R^n$.
- $\|x\| = 0 \; if \; and \; only \; if \; x = 0$.
- $\|ax\| = |a|\|x\|$ for all $a \in R \; and \; x \in R^n$.
- $\|x + y\| \leq \|x\| + \|y\|$ for all $x, y \in R^n$.

Definition 1.15 (Bi- harmonic equation) [14]: A Bi- harmonic equation in two – dimension is defined by:

$$\Delta^2 \boldsymbol{u} = \boldsymbol{F}$$

Where u is a domain define and F is a domain function.

الملخص:

في هذه الرسالة ، تم تعريف مساله كوشي العكسية لمعادلة ثنائية لإبلاسي لبعض المجالات الحلقية وغير الحلقية مع بيانات كوشي المعطاة على جزء من الحدود الخارجية للمجال.تم اقتراح طريقة التجميع غير الشبكية باستخدام توسع متعدد الحدود لحل مساله كوشي العكسية لمشكلة القيمة التفاضلية من الدرجة الرابعة المختلطة ، يتم الحصول على مشكلة مباشرة لحل النظام الخطي وباستخدام خوارزميات CGLS و PCG لحل هذا النظام الخطي.لتوضيح كفاءة الطريقة المقترحة لإيجاد بعض الأمثلة مع حالات مختلفة (متعدد الحدود وغير متعدد الحدود).

تم إثبات الاستقرار العددي للطرق المقترحة من خلال إدخال ضوضاء لبيانات حدود الإدخال.