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CHAPTER ONE

INTRODUCTION AND PRELIMINARIES

Chapter One

Introduction and Preliminaries

1.1 Introduction

A Fuzzy S-act is very useful categories in many applications in mathematics and computer sciences such as Fuzzy automata theory, fuzzy pattern recognition and fuzzy finite state machine.

Literature Review: Kuroki in [19, 20] introduced the theory of fuzzy semigroup and Ahsan in [6] was first introduced the fuzzy action on monoid which open the way for researchers to study the fuzzy acts over semigroups and monoids. Injective and quasi injective $S-$acts were studied in [1,23] after that A.A.Dahash in [1] defined semi injective $S-$act, It is a popularization of $QI-$act [6] and semi injective $R-$modules [2].

The previous definitions have many application in language theory and finite machine state [25], this gave the motivation to study the Fuzzification of injective $S-$act and quasi injective $S-$acts [4, 21].

Inception we mention defintion of fuzzy S-act (if L is $S-$act, and $\lambda: L \rightarrow [0,1]$ be a fuzzy subset on L then the tripl (L, λ_L) is called a fuzzy $S-$act if: $\lambda_L(xs) \geq \lambda_L(x) \forall x \in L, s \in S)$ and thendefinition of fuzzy S-homomorphism of two fuzzy S-act.

Tagreed in [21] studied the concept of Fuzzy Quasi-Injective $S-$act (for short $FQI-$act). (A fuzzy $S-$act (A, λ_A) is said to be $FQI-$act if for each fuzzy subact (B, λ_B) of (A, λ_A) and for each $FS-$homomorphism $f: (B, \lambda_B) \rightarrow (A, \lambda_A)$, there is a $FS-$homomorphism $g: (A, \lambda_A) \rightarrow (A, \lambda_A)$

where g semi injective which is denoted by (FSI −act) and mention some of it's is an extension of f.

Aim of the Study: In this thesis, the fuzzification of properties have been studied and the subacts of FSI −act. .

We also mentioned several applications of FSI −act such as the language of f, and mention the definition of Fuzzy injective S −act which is denoted by (FI −act).

This thesis contains three chapters. In chapter one, the basic concepts are given.

Chapter two contains three sections. In section one, the concept of (FSI −act) is presented many examples to explain definition of FSI −act and a FSI −act was given in term of fuzzy intersection large subact, see Theorem (2.1.3). Second, the FSI −act in terms of fuzzy endomorphism of it's injective envelope, see Theorem (2.1.5) finally, we gave some characteristics of FSI −act.

In section two , we presented the notion of the subact of FSI −act, we gave an example to show that subact of FSI −acts need not be FSI −act thus we investigated some kinds of subacts which inherit the FSI −act property such as fuzzy invariant subact, fuzzy retract subact, fuzzy intersection-large and Fuzzy stable subact, finally, we studied some properties of the subacts of FSI −act and we showed if H is semi injective subact of N and (λ, N) is FSI −act, then there exist FSI −act on H, see Theorem (2.2.13).

In section three of this chapter, we presented the application of FSI −act. We explained the definition of finite state automaton,the definition of language, and definition of fuzzy language. We introduced an example that shows the formation language from finite automata and formation of fuzzy language from language.

In chapter three, we introduced concept Intuitionistic Fuzzy Semi Injective $S-$act, which is denoted by (IFSI $-$act) such that we answered the next question why intuitionistic Fuzzy set? and we mentioned definition of the intuitionistic Fuzzy set (IFS), intuitionistic fuzzy $S-$act (IF $-$act), intuitionistic FS$-$homomorphism (IFS$-$ homomorphism) after that we linked the concept of intuitionistic with our work in section one of chapter two (FSI$-$act) and we gave adefinition of intuitionistic Fuzzy semi injective $S-$act (IFSI $-$S act).We explain this definition by giving a detailed example and proved many theorems shows relationship between IFSI$-$act with a direct sum.

1.2 Semigroups and $S-$acts

In this section, some known definitions and results that will be used in the other chapters have been recalled.

Definition (1.2.1):

A set S with an associative binary operation is called a **semigroup.** The operation $S \times S \to S$ on a semi group S is $(a, b) \to ab, a(bc) = (ab)\,c\ \forall a, b, c \in S,$ every group is a semigroup but the converse is not correct for example (N,+) is semi group but it is not group since it has not an inverse element **[12].**

A monoid is any semigroup has an identity element.

Example(1.2.2):

The set $S = \{x, y, z, d\}$ is a finite set contains four elements, and define the product in the table:

	x	y	z	d
x	x	y	z	d
y	y	z	x	z
z	z	x	y	x
d	y	z	x	z

It is easy to show that S is a semigroup by investigation associative operation on S, the element x is an identity of S, and hence S is a monoid.

Definition (1.2.3):

Asubset N of aset S such that $(S,\circledast)$ is semigroup then $(N,\circledast)$ is called a **subsemigroup** of $(S,*)$ if and only if $(N,*)$ is a semigroup under restriction of the operation of S**[13]**.

Example and Remark (1.2.4)

1. $(Z^+,.)$ is a subsemigroup of $(R,.)$ such that $Z^+ \subseteq R$ and $(Z^+,.)$ is a semigroup.
2. Each subsemigroup of a cyclic inverse semigroup with zero element is cyclic.

Definition (1.2.5):

Let S be a monoid and N be anon-empty set, this set is said to be **S −act** N, if we have $\lambda: S \times N \rightarrow N$, $s.t\ (s,n) \rightarrow sn = \lambda(s,n)$ such that; $s(tn) = (st)n\ \forall\ n \in N$ and $s,t \in S$ if S has an identity element 1 $(1.n = n, \forall\ n \in N)$, then we called N a unitary right S −act and if a semigroup S with zero and the following hold: $0.n = 0\ \forall n \in N$, then we called N is a right S −act with zero element 0**[18]**.

S-act is a generalization of R-module (A module is amathematical object in which things can be added together commutatively by multiplying coefficients and in which most of the rules of manipulating vectors hold

.Amodule is abstractly very similar to avector space, although in modules, coefficients are taken in rings that are much more general algebraic objects than the fields used in vector spaces. A module taking its coefficients in aring R is called a module over R, or a R-module [16]).

Example (1.2.6):

The set $S = \{a, b, c\}$ with product in the following table:

$\cdot$	a	b	c
a	a	a	a
b	a	b	b
c	a	b	c

S is a finite semigroup with identity element c and zero element a, we will check that it is S −act, we must fulfill the condition:

$$h(sk) = (hs)\,k \qquad \forall\, h, s, k \in S.$$

Let $s = a$, $h = b$ and $k = c$, then:

$$b(ac) = (ba)c \qquad b(ca) = (bc)a \qquad c(ab) = (ca)b$$
$$ba = ac \qquad ba = ba \qquad ca = ab$$
$$a = a \qquad a = a \qquad a = a$$

In the same way we check the other elements then S is S −act.

Example (12.7):

We define the product in the table of the finite commutative semigroup with c_0 is zero element and $c_i c_j = c_i \ \forall i \leq j$.

$$(c_0c_1)c_2 = c_0(c_1c_2) \qquad (c_1c_0)c_2 = c_1(c_0c_2) \qquad (c_2c_1)c_0 = c_2(c_1c_0)$$
$$c_0c_2 = c_0c_1 \qquad c_0c_2 = c_1c_0 \qquad c_1c_0 = c_2c_0$$
$$c_0 = c_0 \qquad c_0 = c_0 \qquad c_0 = c_0 \cdots\cdots$$

The finite commutative semigroup is $S-$act.

Definition (1.2.8):

A subact H of an S-act N, denoted $H \subseteq N$ is a nonempty subset $H \leq N$ such that $hs \in H$ for all $h \in H$, and $s \in S$**[18]** .

Definition (1.2.9):

Let H, F be $S-$acts, an **S −homomorphism** $g: H \to F$ is a function of H to F, then $g(hs) = g(h)s$, such that $h \in H$ and $s \in S$ **[18]**.

Definition (1.2.10)[18]:

Let g: N → H be an homomorphism where N,H be two S-acts then:

1. g is a monomorphism, if it $1-1$ mapping.
2. g is epimorphism, if it onto mapping.
3. g is an isomorphism, if it is both a monomorphism and epimorphism.
4. g is an endomorphism if $N = H$.
5. g is an automorphism, if it's both an isomorphism and endomorphism.

The symbol hom (N, H) means the set of all $S-$homomorphism from an $S-$act N to an $S-$act H and End (N) means the function hom (N, N) that called the $S-$endomorphism.

Definition (1.2.11):

An $S-$homomorphism $f: A \rightarrow B$ is said to be **retraction** if there exist an $S-$homomorphism $h: B \rightarrow A$ such that $f \circ h = I_B$ and B said to be retract of A (where A, B are two $S-$act) **[18]**.

In the following we recall a large subact and intersection large and some of their properties that will be needed.

Definition (1.2.12):

Let N be an $S-$act, H a subact of N is said to be **large** (or essential) in N if for any $S-$act F and $S-$homomorphism $g: N \rightarrow F$ which restriction to H is one to one then g is a one to one itself at that time we say that N is an essential extension of H **[8]**.

Definition (1.2.1□):

Anon zero subact H of $S-$act N is **intersection large (**simply $\cap-$large) if every nonzero subact A of N, $A \cap H \neq \emptyset$ and will be denoted by $H \subseteq' N$ **[2□]**.

Example and Remarks (1.2.1□)

1. Each large subact of an $S-$act N is $\cap-$large [11], but the converse is not true as we can see in the following example: let $S = \{e_0, e_1, \cdots, e_n\}$ be a semigroup. We consider S as $S-$act over itself

and $H = \{e_0, e_1, \cdots, e_{n-1}\}$ define $g: S \to S$ by $g(e_n) = e_{n-1}$, $g(e_i) = e_i \ \forall\, i \leq n$ and $n \geq 3$, $g|_H$ is one to one while g is not one to one, thus H is not large in S for any $e_i \in S$, $e_i S^1 = \{e_0, e_1, \cdots, e_i\}$ then $\{e_0, e_1\} \subset e_i S^1 \cap H$ contains more than two elements and so H is $\cap -$large subact of S.

2. Let $D \subseteq \acute{D} \subseteq F$, then D large in F iff D large in D' and D' large in F [8].

□roposition (1.2.1□) [1□]: Let A,B,C be S-acts

1. Let $f \in \mathrm{Hom}(A, B)$ if $C \subseteq' B$, then $f^{-1}(C) \subseteq' A$.
2. If $A_i \subseteq' B$, then $\cap A_i \subseteq' B$, $i = 1,2, \dots \dots \dots n$.
3. If $A \subseteq' B \subseteq C$ and $A' \subseteq' B \subseteq C$, then $A \cap A' \subseteq' B \cap B'$.
4. If $A \subseteq B \subseteq C$ then $A \subseteq' C$ if and only if $A \subseteq' B$ and $B \subseteq' C$.

Definition (1.2.1□): Let H be a subact of an $S-$act N. A relative complement of H in N is any subact Y of N which means that Y is **ma□ima□** with respect to the property $H \cap Y = \emptyset$**[10]**.

□roposition (1.2.1□):

Let H be a subact of the $S-$act N, if Y is any relative complement of H in N, then $H \cup Y \subseteq' N$**[10]**.

Definition (1.2.18):

A subact K of an $S-$act N is called **stable** subact of N if $\alpha(K) \subseteq K$ for every $S-$homomorphism $\alpha: K \to N$ and N is said to be **fully stable** in case each subact of N is stable**[1□]**.

Definition(1.2.19):

$S-$act N is said to be **simple** if it is not contain subact other than N itself **[18]**.

Remark (1.2.20):

Every simple $S-$act is fully stable **[10]**.

Definition (1.1.21):

Let E be $S-$act and $B \subseteq E$, then B is called **fully invariant** if $\alpha(B) \subseteq B$ for each endomorphism α of E and E is called **duo** if every subact of E is fully invariant. For example: $S = (\mathbb{Z}, .)$ if S as $S-$act over itself then S duo-act **[2□]**.

Remark(1.2.22):

Stable subact is invariant but the converse may not be true generally**[10]**. The $Z-$act (Z integer numbers) is not fully stable define $f: 3Z \to Z$ by $f(3z) = 5z \ \forall z \in Z$ clearly f is a $Z-$homomorphism.

$f(az) = f(a)z$ where $a \in 3z$

$f(3.1) = f(3).1 \Rightarrow f(3) = f(3)$

$f(3.2) = f(3).2 \Rightarrow f(3.2) = 10 = f(3).2 = 10$

But $f(3z) \nsubseteq 3z$ since $f(3.2) = 10 \nsubseteq 3z$.

Definition (1.2.2□):

An S −act N is said to be **mu□tip□ication** if every subact of N can be write NI for some ideal I of S. For example $(Z, .)$ is multiplication Z −act.

Now we will define direct sum**[10]**.

Definition (1.2.2□):

The disjoint union $N \dot{\cup} H$ or $(N \oplus H)$ is the **□irect sum** of two S −acts N and H, in the case of being H is subact of N then H is called **□irect summan□** of N and if there exist any subact K of N then $H \cap K = \emptyset$ and $H \cup K = N$**[□1]** .

Definition (1.2.2□) :

A congruence ρ on a S-act N, is an **e□ui□a□ence re□ation** defined on N such that if $a \rho b$ then $(as) \rho (bs)$if and only if that mean $as = bs$ for all $s \in S$ and a,b$\in$ N, we shall denote ρ by Ψ_N.

The congruence Ψ_N on S −act N is said to be singular congruence on N. An S −act N is said to be singular if $\Psi_N = N \times N$ and non-singular if $\Psi_N = i_N$ where i_N is trivial congruence on N (i.e., $x i_N y$ if x=y) **[2□]**.

1.□ Quasi–injective S–act and Semi–injective S–act

In this section the injective, quasi–injective and semi- injective have been reviewed and have been mentioned some concepts that we need to clarify concepts quasi injective and semi injective.

Definition (1.□.1) :

An $S-$act N is called **injective** if for each $S-$monomorphism f from $S-$act K to $S-$act L and each $S-$homomorphism g from K to N there exists an $S-$homomorphism g' from L to N such that $g' \circ f = g$ **[1□]** (see Figure (1.1).

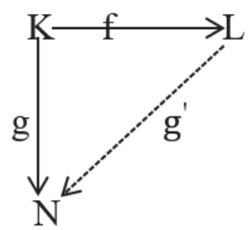

Figure 1.1

Definition(1.□.2):

The injective envelope is **a maximal essential extension** of an $S-$act N. It is denoted by I(N) **[8]**.

* I(N) =N if and only if N is injective .

* If I(N) is an injective S-act containing N, then I is an injective envelope of N if and only if N essential in I .

المستخلص

في هذه الأطروحة قُدِمَ مفهوم الافعال من النمط S ضبابية فقيرة الاغمار ويشار اليها بواسطة FSI-act , وهو يعطي تعميم لكل من الافعال الضبابية الاغمارية(FI – act) والافعال الضبابية شبه اغمارية (FQI-act). اولا اعطينا الحافز لتقديم هذا المفهوم من خلال اعطاء رسم تخطيطي ابدالي يوضح الضبابية للأفعال فقيرة الاغمار مع ذكر تعريف لمفهوم (FSI-act).كما وتم اعطاء وصف للافعال الضبابية فقيرة الاغمار تحت شرط الافعال الجزئية الضبابية الكبيرة وايضا تحت شرط التشاكل الضبابي الذاتي للافعال الأغماريةز ثم تقديم العديد من المقترحات لشرح بعض خصائص الافعال فقيرة الاغمار , على سبيل المثال العلاقة بينهما وبين الافعال الضبابية شبه الاغمارية .ثانيا قُدِمَ مفهوم الافعال من النمط Sضبابية فقيرة الاغمار جزئيةوالمشار لها بالرمز (FSI-subact). واعطينا مثال يبين ان الافعال الضبابية الفقيرة الاغمارية الجزئية من افعال ضبابية فقيرة اغمارية لا تحتاج ان تكون افعال ضبابية اغمارية فقيرة وعليه تحققنا من بعض انواع الافعال الضبابية الاغمارية الجزئية التي ترث صفات الافعال الضبابية الفقيرة الاغمارية وهي الانكماش الضبابي الجزئي, التقاطع الكبير الضبابي الجزئي, الثابت التام الضبابي الجزئي والثابت الضبابي الجزئي .

ودرسنا بعض خواصها وبالنهاية عرفنا الافعال الضبابية الفقيرة الاغمارية الحدسية التي اشرنا اليها بواسطة (IFSI-act) وناقشنا علاقتها مع الجمع المباشر .