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قسم علوم الرياضيات

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مـــن قبـــل

صـــلاح ابراهـــيم محـــمد

بأشـــراف

أ.م .د

فاطمـة محمــد عـبود

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Chapter One

Some basic concepts

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1.1 Introduction

In this chapter, we recall some basic concepts that we need in our work

1.2 Some Basic definitions and properties

Definition 1.2.1 (Well posed and ill-posed problems) [9]: A well-posed problem satisfies the following characteristics:

1. There must be a solution to the problem.

2. The problem's solution must be unique (uniqueness),

3. The problem must rely continually on the supplied data (stability),

When one of these three requirements is not met, the problem is said to be ill-posed.

Definition 1.2.2 (Direct problems) [21]: A well-posed problem with the uniqueness, stability, and existence of a solution to the associated mathematical problem is referred to as a direct problem.

Definition 1.2.3 (Inverse problems) [21]: The problem is referred to be an inverted problem if one of the conditions needed to solve the direct problem is not met.

Definition 1.2.4 (Boundary conditions) [23]: Boundary conditions are the constraints needed to solve a boundary value problem. A boundary value problem is a differential equation whose solution depends on a domain defined on the bounds of a set of conditions

1. Dirichlet boundary conditions.
2. Neumann boundary conditions.
3. Specifying a weighted average of first and second kinds is called Cauchy boundary conditions.
4. Boundary condition for Robin.

Definition 1.2.5 (Inverse Cauchy problems) [1]: When the boundary conditions for the solution with the normal derivative are only specified on a portion of the boundary of the solution domain and no data is provided on the remaining portion of the boundary, the problem is referred to as a Cauchy problem.

Definition 1.2.6 (Instability) [21]: The instability of the solution means that a small measurement error in the input data can greatly amplify the errors in the solution

Definition 1.2.7 (Smooth boundary) [28]: The border of a domain is said to have a smooth boundary if we can identify a function that is continuous with all of its derivatives on the boundary and its closure.

Definition 1.2.8 (Laplacian operator) [8]: The Laplacian operator is defined as follows.

$$\Delta\, t = \nabla . \nabla\, \mathrm{t} = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2}$$

The gradient of t is define by

$$grad\ t \equiv \nabla\, \mathrm{t} = \frac{\partial \mathrm{t}}{\partial x}\vec{\imath} + \frac{\partial \mathrm{t}}{\partial y}\vec{\jmath}$$

Definition 1.2.9 (Helmholtz equation) [2]: The Helmholtz equation is given by $\Delta t\ +\ k^2\ t = F$

Definition 1.2.10 (Normal derivative) [26]: The normal derivative is a direction derivative that is outward normal (orthogonal) to a curve, surface, or hyper surface and is calculated by the inner product of the gradient and the normal vector.

$$\frac{\partial \mathrm{t}}{\partial \mathrm{n}} = \nabla \mathrm{t}.\vec{\mathrm{n}}$$

Definition 1.2.11 (Inner product) [26]: The inner product of a vector $x\ = (x_1, \cdot\cdot\cdot, x_n) \in R^n$ by a vector $y\ = (y_1\ , \cdot\cdot\cdot, y_n) \in R^n$, is defined by

$$< x, y >= x^T . y = x_1 y_1 + \cdots + x_n y_n.$$

Definition 1.2.12 (A norm of a vector) [26]: A norm of a vector x, denoted by $\|x\|$, is a real function with the properties

- $\|x\| \geq 0$
- $\|x\| = 0 \quad if\ and\ only\ if\ x = 0$
- $\|x + y\| \leq \|x\| + \|y\|$
- $\|\alpha x\| = |\alpha| \|x\|$

Where x and y are vectors and for any scalar α

Definition 1.2.13 (Symmetric and positive definite matrices) [26]: If $\mathrm{A} = \mathrm{A}^{\mathrm{T}}$, or if a square matrix A of order n equals its transpose, the matrix is said to be symmetric; these matrices are known as positive definite matrices. A square matrix A is positive definite if

$$x^T A\, x > 0, \quad x \neq 0$$

Definition 1.2.14 (Meshless Methods) [18]: The meshless approach is an alternative to the mesh-based method (finite difference or volume, finite element, boundary element, etc.). The meshless approach uses node points inside a problem domain without requiring nodal connection.

Definition 1.2.15 (Errors) [26]: There are two well-known measures of errors which are the absolute error and the relative error:

- The absolute error is defined by the absolute value of the difference between the approximate (calculated) solution and the exact (actual) solution.
- The relative is defined by the division of the absolute error by the norm of the exact solution.

Definition 1.2.16 (Tolerance) [25]: The Tolerance, abbreviated by Tol, is the programmer's default choice for admissible error, which is typically close to zero.

Definition 1.2.17 (Condition number) [26]: The condition number is defined by the product of the norm of a matrix by the norm of its inverse:

$$\kappa(A) = \|A^{-1}\| \, \|A\|$$

The matrix A need to have an inverse.

Recalling that a problem is said to be well-conditioned if it has a small condition number and is said to be ill-conditioned if it has a large condition number.

Definition 1.2.18 (Preconditioning) [26]: Preconditioning is a numerical method for ill-conditioned problems that involves multiplying the issue by a preconditioned matrix, which has the benefit of lowering the condition number of the corresponding matrix for the problem.

خلاصة:

في هذا العمل تم حل مشكلة كوشي العكسية المعرفة على مجال ثنائي الأبعاد تشغله مادة تحقق معادلات نوع هلمهولتز، وذلك بالاستفادة من بيانات كوشي الإضافية على الجزء الذي يمكن الوصول إليه من الحدود.

إن تقريب الحل عن طريق توسيع متعدد الحدود يؤدي إلى حل مشكلة مباشرة لمعادلة النظام الخطي، وهي طريقة بدون شبكة.

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تم التحقق من استقرار الطريقة من خلال تطبيق معلمات الضوضاء المختلفة.