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وهي كجزء من متطلبات نيل درجة الماجستير في علوم الرياضيات

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Chapter one

Some fundamental properties of fuzzy set and basic definitions

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Some fundamental properties of fuzzy set and basic definitions

1-1 Introduction:

In this chapter, we study three section, we recall some basic concept that we need in our work. and we are going to introduce some types of fuzzy closed set and fuzzy open sets in a fuzzy topological space such a fuzzy closed set (f.c.s), fuzzy regular closed set (f.r.c.s) fuzzy $g^* -$ closed set (f. g^*.c.s),fuzzy g – closed set (f.g.c.s), fuzzy rg – closed set (f.rg.c.s) and study the relation among them and give some propositions which support our search, also we are going to define fuzzy regular open set (f.r.o.s), fuzzy g – open and study the relation among them. We also introduce the concepts of fuzzy $\alpha -$ open set, fuzzy $\alpha -$ closed set, fuzzy $\beta -$ open set, fuzzy $\beta -$ closed set, fuzzy pre – open set, fuzzy pre – closed set. We examine fundamental characteristics of these sets using fundamental theorems.

1−2 Basic concepts of fuzzy set

In this section, we recall some basic concept that we need in our work.

Definition (1.2.1)[53] :- let $\widetilde{X}$ be a non – empty set a fuzzy set $\widetilde{A}$ in $\widetilde{X}$ is characteristic by a membership function $\mu\widetilde{A} : \widetilde{X} \to [0,1]$ and we can write this fuzzy set as:

$\widetilde{A} = \{ (x , \mu\widetilde{A}(x) : x \in X , \mu\widetilde{A}(x) \leq 1\}$

The collection of all fuzzy sets in $\widetilde{X}$ will be denoted by I^x , when $I^x = \{ \widetilde{A}: \widetilde{A}$ is fuzzy set in $\widetilde{X}\}$.

Definition (1.2.2)[53] :- The support of a fuzzy set $\widetilde{A}$ is the set of all $x \in X$ such that $\mu\widetilde{A}(x) > 0$ and is denoted by $S(\widetilde{A})$.

Definition (1.2.3)[53] :- A fuzzy point $\widetilde{P}_x^r$ in $\widetilde{X}$ is a special fuzzy set with membership function defined by $\widetilde{P}_x^r(y) = \begin{cases} r, \text{if } x = y \\ 0, \text{if } x \neq y \end{cases}$

When $0 < r \leq 1$, y is the support of $\widetilde{P}_x^r(x)$.

Definition (1.2.4) [53]:- A fuzzy set $\widetilde{A}$ is said to be finite fuzzy set if $S(\widetilde{A})$ is a finite set.

Remark (1.2.5):-

1) A non –empty set X is a fuzzy set with membership $\mu\widetilde{X}(x) = 1, \forall x \in X$

and X is called a crisp set.

2) A membership function $\mu\widetilde{\emptyset}(x) = 0, \forall x \in X$ is called an empty set and denoted by $\widetilde{\emptyset}$.

Definition (1.2.6) [53]:- Let $\widetilde{P}_x^r$ be a fuzzy point and $\widetilde{C}$ be a fuzzy set in non-empty set $\widetilde{X}$. Then $\widetilde{P}_x^r$ is said to be in $\widetilde{C}$ or $\widetilde{C}$ contains $\widetilde{P}_x^r$ if $\mu\widetilde{P}_x^r \leq \mu\widetilde{C}(x)$ for all $x \in X$ and denoted by $x \in S(\widetilde{C})$.

Definition (1-2-7) [53] :- Let $\widetilde{A}$ and $\widetilde{B}$ be fuzzy sets of a universal set $\widetilde{X}$ then

1) $\widetilde{A} \subseteq \widetilde{B}$ if and only if $\mu\widetilde{A}(x) \leq \mu\widetilde{B}(x), \forall x \in X$

2) $\widetilde{A} = \widetilde{B}$ if and only if $\mu\widetilde{A}(x) = \mu\widetilde{B}(x)$ for all $x \in X$

3) $\widetilde{A}^C$ is the complement of a fuzzy set $\widetilde{A}$ with membership function

$\mu\widetilde{A}^C = 1 - \mu\widetilde{A}(x)$

4) $\tilde{C} = \tilde{A} \cup \tilde{B}$ if and only if $\mu\tilde{C}(x) = \max\{\mu\tilde{A}(x), \mu\tilde{B}(x)\}$, $\forall\ x \in X$.

5) $\tilde{D} = \tilde{A} \cap \tilde{B}$ if and only if $\mu\tilde{D}(x) = \min\{\mu\tilde{A}(x), \mu\tilde{B}(x)\}$, $\forall\ x \in X$.

6) More generally, for a family of fuzzy sets { $\tilde{A}_\alpha$: $\alpha \in \wedge$ where $\wedge$ is the any index set }

the union $\tilde{C} = \bigcup_{\alpha\in\wedge} A\alpha$ and the intersection $\tilde{D} = \bigcap_{\alpha\in\wedge} A\alpha$ are defined respectively by

$\mu\tilde{C}(x) = \sup_{\alpha\in\wedge} \{\mu\tilde{A}(x); x \in X\}$

$\mu\tilde{D}(x) = \inf_{\alpha\in\wedge} \{\mu\tilde{A}(x); x \in X\}$

Proposition (1.2.8) [53] :- Let $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ be fuzzy sets in X, the following properties are satisfied:

1) Commutatively: $\max\{\mu\tilde{A}(x), \mu\tilde{B}(x)\} = \max\{\mu\tilde{B}(x), \mu\tilde{A}(x)\}$ and $\min\{\mu\tilde{A}(x), \mu\tilde{B}(x)\} = \min\{\mu\tilde{B}(x), \mu\tilde{A}(x)\}$.

2) Associativity: $\max\{\mu\tilde{C}(x), \max\{\mu\tilde{A}(x), \mu\tilde{B}(x)\}\} = \max\{\mu\tilde{A}(x), \max\{\mu\tilde{B}(x), \mu\tilde{C}(x)\}\}$.

3) Idempotent : $\mu\tilde{A}(x) = \max\{\mu\tilde{A}(x), \mu\tilde{A}(x)\}$, $\mu_{\tilde{A}}(x) = \min\{\mu\tilde{A}(x), \mu\tilde{A}(x)\}$.

4) Distributive : $\max\{\mu\tilde{A}(x), \min\{\mu\tilde{B}(x), \mu\tilde{C}(x)\}\} = \min\{\max\{\mu\tilde{A}(x), \mu\tilde{B}(x)\}, \max\{\mu\tilde{A}(x), \mu\tilde{C}(x)\}\}$ and $\min\{\mu\tilde{A}(x), \max\{\mu\tilde{B}(x), \mu\tilde{C}(x)\}\} = \max\{\min\{\mu\tilde{A}(x), \mu\tilde{B}(x)\}, \min\{\mu\tilde{A}(x), \mu\tilde{C}(x)\}\}$.

5) $\min\{\mu\tilde{A}(x), \mu\tilde{\emptyset}(x)\} = \mu\tilde{\emptyset}(x)$, $\max\{\mu\tilde{A}(x), \mu\tilde{X}(x)\} = \mu\tilde{X}(x)$

6) Identity : $\max\{\mu\tilde{A}(x), \mu\tilde{X}(x)\} = \mu\tilde{X}(x)$

7) $\max\{\mu\tilde{A}(x), \min\{\mu\tilde{A}(x), \mu\tilde{B}(x)\}\} = \mu\tilde{A}(x)$

8) De Morgan law : $(\max\{\mu\tilde{A}(x), \mu\tilde{B}(x)\})^C = \min\{\mu\tilde{A}^C(x)\ ,\ \mu\tilde{B}^C(x)\}$

$(\min\{\mu\tilde{A}(x), \mu\tilde{B}(x)\})^C = \max\{\mu\tilde{A}^C(x)\ ,\ \mu\tilde{B}^C(x)\}$

9) $(\mu\tilde{A}^C(x))^C = \mu\tilde{A}(x)$.

Definition (1.2.9)[8] :- A collection $\tilde{T}$ on a fuzzy set $\tilde{A}$, such that $\tilde{T} \subseteq P(\tilde{A})$ is said to be a fuzzy topology on a fuzzy set $\tilde{A}$ if it satisfied the following conditions:

1 - $\tilde{A}$, $\tilde{\emptyset} \in \tilde{T}$.

2 - The intersection of finite members of fuzzy sets of $\tilde{T}$ is a member of $\tilde{T}$.

3 - The union of any members of $\tilde{T}$ is a member of $\tilde{T}$.

Remark (1.2.10) :-

1 - The members of $\tilde{T}$ are called fuzzy open sets and are denoted by f.o.s.

2 - If $\tilde{B} \in \tilde{T}$, the complement of $\tilde{B}$ ($\tilde{B}^C$) is called fuzzy closed set (briefly f.c.s) and defined as $\mu\tilde{B}^C(x) = \mu\tilde{A}(x) - \mu\tilde{B}(x)$, $\forall\ x \in X$, $\tilde{A}$ is the universal fuzzy set on which $\tilde{T}$ is defined .

Definition(1.2.11)[8]:- Let $(\tilde{A}, \tilde{T})$ be f.t.sp and let $\tilde{B}$ be afuzzy set in $(\tilde{A}, \tilde{T})$, then the closure of $\tilde{B}$ (denoted by $\overline{\tilde{B}}$) and the interior of $\tilde{B}$ (denoted by $\tilde{B}^o$) are defined respectively by :

$\overline{\tilde{B}} = \cap\ (\tilde{H}:\tilde{B} \subseteq \tilde{H}, \tilde{H}^c \in \tilde{T}\}$

$\tilde{B}^o = \cup\ \{\tilde{K}:\tilde{K} \subseteq \tilde{B}, \tilde{K} \in \tilde{T}\ )$

Definition (1.2.12)[34]:- Let $(\widetilde{A}, \widetilde{T})$ be f.t.sp then a fuzzy set $\widetilde{B}$ is said to be a fuzzy neighborhood of a fuzzy point $\widetilde{P}_x^r$ if there exist a f.o.s $\widetilde{H}$ in $(\widetilde{A}, \widetilde{T})$ such that $\widetilde{P}_x^r(x) \leq \mu\widetilde{H}(x) \leq \mu\widetilde{B}(x)$.

Example (1.2.13):- Let X = { a ,b ,c } and $\widetilde{A}$ ={(a ,0.8) ,(b, 0.7) ,(c ,0.4)}

Then

$\widetilde{T}$= { $\widetilde{A}$, $\widetilde{\emptyset}$,{(a ,0.4) ,(b ,0.3) ,(c ,0.2)} ,{(a ,0.3) ,(b ,0.1) , (c ,0.1)} is a f.t.sp on $\widetilde{A}$.

Proposition (1.2.14)[8]:- Let $(\widetilde{A}, \widetilde{T})$ be a f.t.sp, let $\widetilde{B}, \widetilde{C}$ be two fuzzy sets in $(\widetilde{A}, \widetilde{T})$ then for all $x \in X$.

1 - $\mu\overline{\widetilde{\emptyset}} = \mu\widetilde{\emptyset}\,(x)$.

2 - $\overline{\widetilde{A}}$ is a f.c.s .

3 – If $\widetilde{H}$ is a f.c.s , such that $\mu\widetilde{B}(x) \leq \mu\widetilde{H}(x)$ where $\widetilde{B}$ is a fuzzy set then $\mu\widetilde{B}(x) \leq \mu\overline{\widetilde{B}}(x) \leq \mu\widetilde{H}(x)$

4 - $\widetilde{B}$ is a f.c.s if and only if $\mu\widetilde{B}(x) = \mu\overline{\widetilde{B}}(x)$

5 – $\left(\overline{\max\{\mu\widetilde{B}(x), \mu\widetilde{C}(x)\}}\right) = \max\{\mu\overline{\widetilde{B}}(x), \mu\overline{\widetilde{C}}(x)\}$

6 - $\mu\overline{\overline{\widetilde{B}}} = \mu\overline{\widetilde{B}}(x)$

7 – If $\mu\widetilde{B}(x) \leq \mu\widetilde{C}(x)$, then $\mu\overline{\widetilde{B}}(x) \leq \mu\overline{\widetilde{C}}(x)$

8 - $\left(\overline{\min\{\mu\widetilde{B}(x), \mu\widetilde{C}(x)\}}\right) \leq \min\{\mu\overline{\widetilde{B}}(x), \mu\overline{\widetilde{C}}(x)\}$

Definition (1.2.15):- Let $\mu\widetilde{A}_1(x), \mu\widetilde{A}_2(x), \ldots, \mu\widetilde{A}_n(x)$ be the memberships of $\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_n$ repectively . Then for all $x_1 \in X_1,\ x_2 \in X_2,\ \ldots, x_n \in X_n$, the Cartesian product

$\widetilde{A}_1 \, X \, \widetilde{A}_2 X \ldots X \, \widetilde{A}_n$ is $\mu\widetilde{A}_1 \, X \, \widetilde{A}_2 X \ldots X \, \widetilde{A}_n(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n) =$

$\min \{ \mu\widetilde{A}_1(x), \mu\widetilde{A}_2(x), \ldots, \mu\widetilde{A}_n(x)\}$.

Definition (1.2.16) :- Let X be the Cartesian product of universes $X_1, X_2, \ldots, X_r$ which is denoted by X and $\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_r$ be r-fuzzy subsets in $X_1, X_2, \ldots, X_r$ respectively with cartesion product $\widetilde{A} = \widetilde{A}_1 \, X \, \widetilde{A}_2 X \ldots X \, \widetilde{A}_r$ and f is a function from X to a universe Y

($y = f(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$). Then the extension principle allows to define a fuzzy subset

$\widetilde{B} = f(\widetilde{A})$ in y by

$\widetilde{B} = \{(y, \mu B(y)) : y = f(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_r), (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_r) \in X\}$

Where

$$\mu\widetilde{B}(y) = \begin{cases} \sup\min\left\{\mu\widetilde{A}_1(x_1), \ldots, \mu\widetilde{A}_r(x_r)\right\} \text{ if } f^{-1} \neq \widetilde{\emptyset} \\ \qquad (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_r) \in f^{-1}(y) \\ 0 \qquad\qquad \text{otherwise} \end{cases}$$

Where f^{-1} is the inverse image of f.

For r =1, the extension principle reduces to

$\widetilde{B} = f(\widetilde{A}) = \{(y, \mu\widetilde{B}(y)) : y = f(x), x \in \widetilde{X}\}$ where

$$\mu\widetilde{B}(y) = \begin{cases} \sup_{\alpha \in f^{-1}(y)} \mu\widetilde{A}(x) \text{ if } f^{-1}(y) \neq \emptyset \\ 0 \qquad \text{other wise} \end{cases}$$

Also the inverse image of fuzzy set is defined as follows $\mu f^{-1}(\widetilde{B})(x) = \mu\widetilde{B}(f(x))$ for all $\tilde{x} \in \widetilde{X}$ and $\widetilde{B}$ fuzzy set of Y.

Definition (1.2.17)[8]:- Let $(\widetilde{X}, \widetilde{T})$ be a f.t.sp and let $\tilde{C}$ and $\widetilde{D}$ be fuzzy sets in $(\widetilde{X}, \widetilde{T})$ then:-

1 – A fuzzy point $\widetilde{P}_x^r$ is said to be quasi coincident with a fuzzy set $\tilde{C}$ if there exist $\tilde{x} \in \widetilde{X}$ such that $\mu\widetilde{P}_x^r + \mu\tilde{C}(x) > \mu\widetilde{X}(x)$ and which is denoted by $(\widetilde{P}_x^r \; q \; \tilde{C})$. If $\widetilde{P}_x^r + \mu\tilde{C}(x) \leq \mu\widetilde{X}(x)$ for all $\tilde{x} \in \widetilde{X}$ the $\widetilde{P}_x^r$ is called not quasi coincident with a fuzzy set $\tilde{C}$ and which is denoted by $(\widetilde{P}_x^r \; \hat{q} \; \tilde{C})$.

2 - A fuzzy set $\tilde{C}$ is said to be quasi coincident with a fuzzy set $\widetilde{D}$ if there exist $\tilde{x} \in \widetilde{X}$, such that $\mu\tilde{C}(x) + \mu\widetilde{D}(x) > \mu\widetilde{X}(x)$ and which is denoted by $(\tilde{C} \; q \; \widetilde{D})$and if $\mu\tilde{C}(x) + \mu\widetilde{D}(x) \leq \mu\widetilde{X}(x)$ for all $\tilde{x} \in \widetilde{X}$ then $\tilde{C}$ is not quasi coincident with a fuzzy set $\widetilde{D}$ which is denoted by $(\tilde{C} \; \hat{q} \; \widetilde{D})$.

Definition (1.2.18)[8]:- A fuzzy set $\widetilde{A}$ of a f.t.sp $(\widetilde{X}, \widetilde{T})$ is called a fuzzy proper if $\widetilde{A} \neq \widetilde{\emptyset}$ and $\widetilde{A} \neq \widetilde{X}$.

1-3 Certain types of fuzzy g – closed sets and fuzzy g – open sets

In this section , we are going to introduce some types of fuzzy closed set and fuzzy open sets in a fuzzy topological space such a fuzzy closed set (f.c.s), fuzzy regular closed set (f.r.c.s) fuzzy $g^* -$ closed set (f. g^*.c.s), fuzzy g – closed set (f.g.c.s), fuzzy rg – closed set (f.rg.c.s) and study the relation among them and give some propositions which support our search, also we are going to define fuzzy regular open set (f.r.o.s) , fuzzy g – open set (f.g.o.s) and study the relation among them.

Definition(1.3.1):- Let $(\widetilde{X}, \widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a fuzzy set in $(\widetilde{X}, \widetilde{T})$, then $\widetilde{B}$ is called a f.r.c.s if $\mu\widetilde{B}(x) = \mu\overline{\widetilde{B}^{o}}(x)$.

Example (1.3.2):- $\widetilde{X}$={(a,0.9) ,(b,0.9) ,(c,0.9)}

$\widetilde{T}$={$\widetilde{\emptyset}$,$\widetilde{X}$,{(a,0.9) ,(b,0.0) ,(c,0.0)} ,{(a,0.0) ,(b,0.9) ,(c,0.9)}}

$\widetilde{A}$= {(a,0.9) ,(b,0.0) ,(c,0.0)} then $\widetilde{A}$ is a f.r.c.s.

Proposition(1.3.3)[47]:- Every f.r.c.s is a f.c.s

Proof:- It is clear.

Remark (1.3.4):- The converse of proposition (1.3.3) need not be true as the following example :-

Example (1.3.5):- Let $\widetilde{X}$={(a,0.7) ,(b,0.7) ,(c,0.7)}

$\widetilde{T}$={$\widetilde{\emptyset}$,$\widetilde{X}$,{(a,0.7) ,(b,0.0) ,(c,0.0)}} then $\widetilde{A}$= {(a,0.0) ,(b,0.7) ,(c,0.7)} is fuzzy closed (f.c.s) but is not f.r.c.s.

Definition (1.3.6):- Let $(\widetilde{X}, \widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a fuzzy set in $(\widetilde{X}, \widetilde{T})$, the $\widetilde{B}$ is called a f.r.o.s if $\mu\widetilde{B}(x) = \mu\overline{\overline{\widetilde{B}}}^{o}(x)$.

Remark (1.3.7)[2]:- Every f.r.o.s is a f.o.s.

Proposition(1.3.8):- Let $(\widetilde{X}, \widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a f.o.s in $(\widetilde{X}, \widetilde{T})$, then $\overline{\widetilde{B}}^{o}$ is a f.r.o.s in $(\widetilde{X}, \widetilde{T})$.

Proof:- Let $\widetilde{B}$ be a f.o.s in $(\widetilde{X}, \widetilde{T})$ First we are going to show that $\mu\overline{\widetilde{B}}^{o}(x) \leq \mu\overline{\overline{\widetilde{B}}^{o}}^{o}(x)$. Now since $\mu\widetilde{B}(x) \leq \mu\overline{\widetilde{B}}(x)$, the $\mu\widetilde{B}^{o}(x) \leq \mu\overline{\widetilde{B}}^{o}(x)$ But $\widetilde{B}$ is a f.o.s in $(\widetilde{X}, \widetilde{T})$ (by hypothesis).

Then $\mu\widetilde{B}^{o}(x) = \mu\widetilde{B}(x)$, so $\mu\widetilde{B}(x) \leq \mu\overline{\widetilde{B}}^{o}(x)$ and $\mu\overline{\widetilde{B}}(x) \leq \mu\overline{\overline{\widetilde{B}}^{o}}(x)$,thus $\mu\overline{\widetilde{B}}^{o}(x) \leq \mu\overline{\overline{\widetilde{B}}^{o}}^{o}(x)$.

Now we are going to show that $\mu\overline{\overline{\widetilde{B}}^{o}}^{o}(x) \leq \mu\overline{\widetilde{B}}^{o}(x)$ since $\mu\overline{\widetilde{B}}^{o}(x) \leq \mu\overline{\widetilde{B}}(x)$, the $\mu\overline{\overline{\widetilde{B}}^{o}}(x) \leq \mu\overline{\overline{\widetilde{B}}}(x)$. But $\mu\overline{\overline{\widetilde{B}}}(x) = \mu\overline{\widetilde{B}}(x)$ Then $\mu\overline{\overline{\widetilde{B}}^{o}}(x) \leq \mu\overline{\widetilde{B}}^{o}(x)$ Hence $\mu\overline{\overline{\widetilde{B}}^{o}}^{o}(x) \leq \mu\overline{\widetilde{B}}^{o}(x)$.

There for $\mu\overline{\widetilde{B}}^{o}(x) = \mu\overline{\overline{\widetilde{B}}^{o}}^{o}(x)$ and thus $\overline{\widetilde{B}}^{o}$ is a f.r.o.s in $(\widetilde{X}, \widetilde{T})$.

Proposition(1.3.9):- Let $(\widetilde{X}, \widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ and $\tilde{C}$ be f.o.s in $(\widetilde{X}, \widetilde{T})$ such that $\min\{\mu\widetilde{B}(x), \mu\tilde{C}(x)\} = 0$ then $\min\{\mu\overline{\widetilde{B}}^{o}(x), \mu\overline{\tilde{C}}^{o}(x)\} = 0$.

Proof :- Let $\widetilde{B}$ and $\tilde{C}$ be f.o.s in $(\widetilde{X}, \widetilde{T})$ such that $\min\{\mu\widetilde{B}(x), \mu\tilde{C}(x)\} = 0$ Thus $\mu\widetilde{B}(x) \leq \mu\tilde{C}^{c}(x)$ and $\mu\tilde{C}(x) \leq \mu\tilde{C}^{c}(x)$.

So $\mu\overline{\widetilde{B}}(x) \leq \mu\tilde{C}^{c}(x)$ and $\mu\overline{\tilde{C}}(x) \leq \mu\widetilde{B}^{c}(x)$ since $\mu\overline{\widetilde{B}}^{o}(x) \leq \mu\overline{\widetilde{B}}(x)$ and $\mu\overline{\widetilde{B}}^{o}(x) \leq \mu\overline{\tilde{C}}(x)$.

Then $\mu\overline{\widetilde{B}}^{o}(x) \leq \mu\overline{\widetilde{B}}(x) \leq \mu\tilde{C}^{c}(x)$ and $\mu\overline{\tilde{C}}^{o}(x) \leq \mu\overline{\tilde{C}}(x) \leq \mu\widetilde{B}^{c}(x)$.

Thus $\min\{\mu\overline{\widetilde{B}}^{o}(x), \mu\overline{\tilde{C}}^{o}(x)\} = 0$.

Definition (1.3.10)[12]:- Let $(\widetilde{X},\widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a fuzzy set in $(\widetilde{X},\widetilde{T})$ then $\widetilde{B}$ is said to be a fuzzy generalized closed set (f.g.c.s) if $\mu\overline{\widetilde{B}}(x) \leq \mu\widetilde{O}(x)$ when ever $\mu\widetilde{B}(x) \leq \mu\widetilde{O}(x)$, where $\widetilde{O}$ is f.o.s in $(\widetilde{X},\widetilde{T})$.

Example(1.3.11):- $\widetilde{A} = \{(1,0.6)\ ,(2,0.6)\}$ be a fuzzy set then $\widetilde{T} = \{\ \widetilde{\emptyset}\ ,\widetilde{A}\ ,\ \{(1,0.3)\ ,(2,0.3)\}$ is a f.t.s on $\widetilde{A}$, take $\widetilde{B} = \{(1,0.2),(2,0.1)\}$ is a fuzzy set of $(\widetilde{A},\widetilde{T})$.

Then $\widetilde{B}$ is a f.g.c.s since the f.o.s which contains $\widetilde{B}$ is $\widetilde{A}$ and contains $\overline{\widetilde{B}}$ too.

Proposition(1.3.12)[12]:- Every f.c.s is a f.g.c.s.

Proof :- It is clear.

Remark (1.3.13):- The converse of proposition (1.3.12) need not be true. In example (1.3.11) $\widetilde{B}$ is a f.g.c.s but is not f.c.s.

Proposition(1.3.14)[47]:- Every f.r.c.s is a f.g.c.s.

Definition(1.3.15):- Let $(\widetilde{X},\widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a fuzzy set of $(\widetilde{X},\widetilde{T})$ then $\widetilde{B}$ is said to be a fuzzy generalized open set (f.g.o.s) if $\widetilde{B}^c$ is a f.g.c.s.

Proposition(1.3.16)[14]:- Every f.o.s is a f.g.o.s.

Remark (1.3.17):- The convers of proposition (1.3.16) need not be true as the following example.

Example (1.3.18):- In the example (1.3.11) take $\widetilde{E} = \{(1,0.4),(2,0.5)\}$ since $\widetilde{E}^c$is a f.g.c.s , then $\widetilde{E}$ is a f.g.o.s clearly $\widetilde{E}$ is not f.o.s.

Proposition(1.3.19)[48]:- Let $(\widetilde{X},\widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a fuzzy set of $(\widetilde{X},\widetilde{T})$ then $\mu(\widetilde{B}^o(x))^c = \mu\overline{\widetilde{B}^c}(x)$.

Theorem (1.3.20)[12]:- Let $(\widetilde{X},\widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a fuzzy set in $(\widetilde{X},\widetilde{T})$ then $\widetilde{B}$ is a f.o.s if and only if $\mu\widetilde{G}(x) \leq \mu\widetilde{B}^{o}(x)$ whenever $\mu\widetilde{G}(x) \leq \mu\widetilde{B}(x)$, where $\widetilde{G}$ is a f.g.c.s in $(\widetilde{A},\widetilde{T})$.

Proof:- Depending on the proposition (1.3.19).

Definition(1.3.21):- Let $(\widetilde{X},\widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a fuzzy set in $(\widetilde{X},\widetilde{T})$ then $\widetilde{B}$ is said to be fuzzy $g^{*}-$ closed set (f. g^{*}.c.s) if $\mu\overline{\widetilde{B}}(x) \leq \mu\widetilde{U}(x)$, whenever $\mu\widetilde{B}(x) \leq \mu\widetilde{U}(x)$, where $\widetilde{U}$ is a f.g.o.s in $(\widetilde{X},\widetilde{T})$, and a fuzzy set $\widetilde{C}$ of $(\widetilde{X},\widetilde{T})$ is said to be a fuzzy g^{*}- open set (f. g^{*}.o.s) if $\widetilde{C}^{c}$ is a f. g^{*}.c.s.

Proposition(1.3.22)[30]:- Every f. g^{*}.c.s is a f.g.c.s.

Proposition(1.3.23)[30]:- Every f.c.s is a f. g^{*}.c.s.

Remark (1.3.24):- The converse of proposition (1.3.23) need not be true as the following example.

Example (1.3.25):- Let $\widetilde{X}$ = {(1,0.7) ,(3,0.2)} be fuzzy set, $\widetilde{T}$ = {$\widetilde{\emptyset}$, $\widetilde{X}$,{(1 ,0.7) ,(3 ,0.0)} is a f.t.s on $\widetilde{X}$.

Take $\widetilde{B}$ ={(1 ,0.3), (3 ,0.5)} be a fuzzy set of $(\widetilde{X},\widetilde{T})$. The f.g.o.s which contains $\widetilde{B}$ is $\widetilde{X}$ and also contains $\overline{\widetilde{B}}$ too, there for $\widetilde{B}$ is f. g^{*}.c.s, clearly $\widetilde{B}$ is not f.c.s.

Proposition(1.3.26):- Let $(\widetilde{X},\widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a f.g.o.s and f.g^{*}.c.s in $(\widetilde{X},\widetilde{T})$ then $\widetilde{B}$ is a f.c.s.

Proof :- Let $(\widetilde{X},\widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a f.g.o.s and f. g^{*}.c.s in $(\widetilde{X},\widetilde{T})$. We are going to show that $\widetilde{B}$ is a f.c.s.

Now $\mu\widetilde{B}(x) \leq \mu\widetilde{B}(x)$ and $\mu\overline{\widetilde{B}}(x) \leq \mu\widetilde{B}(x)$.

But $\mu\widetilde{B}(x) \leq \mu\overline{\widetilde{B}}(x)$, then $\mu\overline{\widetilde{B}}(x) = \mu\widetilde{B}(x)$.

Therefore $\widetilde{B}$ is a f.c.s.

Definition (1.3.27):- Let $(\widetilde{X}, \widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a fuzzy set in $(\widetilde{X}, \widetilde{T})$, then $\widetilde{B}$ is said to be fuzzy regular generalized closed set (f.rg.c.s) if $\mu\overline{\widetilde{B}}(x) \leq \mu\widetilde{U}(x)$, when ever $\mu\widetilde{B}(x) \leq \mu\widetilde{U}(x)$ where $\widetilde{U}$ is a f.r.o.s in $(\widetilde{X}, \widetilde{T})$.

Proposition(1.3.28)[12]:- Every f.g.c.s is a f.rg.c.s.

Corollaries (1.3.29)[47]:- Let $(\widetilde{X}, \widetilde{T})$ be a f.t.s then

1 – Every f.c.s is a f.rg.c.s.

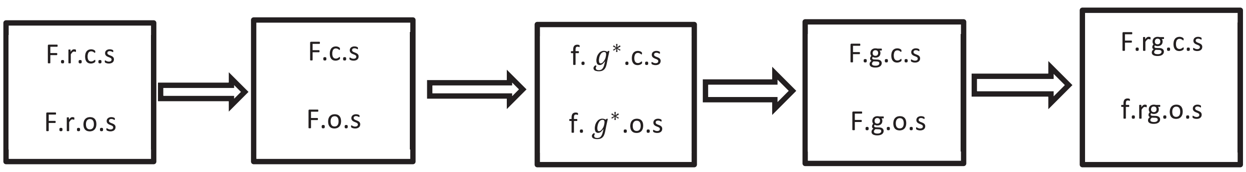
2 – Every f.r.c.s is a f.rg.c.s.

Definition (1.3.30):- Let $(\widetilde{X}, \widetilde{T})$ be a f.t.sp and let $\widetilde{B}$ be a fuzzy set in $(\widetilde{X}, \widetilde{T})$, then $\widetilde{B}$ is said to be f.rg.o.s if $\widetilde{B}^c$ is a f.rg.c.s in $(\widetilde{X}, \widetilde{T})$.

Proposition (1.3.31)[2]:- Every f.g.o.s is a f.rg.o.s.

Corollary (1.3.32)[2]:- Every f.o.s is a f.rg.o.s.

The following diagram illustrate the relation among fuzzy closed sets on the one hand and fuzzy open sets on the other



(Figure 1)

المستخلص

في هذه الرسالة قمنا بدراسة الفضاءات المتصلة الضبابية والتي تم تعريفها على مجموعة ضبابية فضاء متصل مسبقاً ضبابي وفضاء متصل الفا ضبابي و فضاء متصل معمم ضبابي وفضاء متصل *معممة ضبابي وفضاء متصل معمم ومنتظم ضبابي وفضاء متصل بيتا ضبابي وفضاء شبه متصل ضبابي وفضاء متصل مسبقاً ضبابي مع بعض النظريات الأساسية والعلاقات التي تتعلق بأطروحات الفضاء الضبابي.

قمنا بتقديم ودراسة العديد من المجموعات الضبابية المنفصلة والتي تمثل اكثر المفاهيم الأساسية لبناء الفضاءات المتصلة التي قمنا بدراستها مثل مجموعة منفصلة معممة ضبابية و مجموعة منفصلة الفا ضبابية و مجموعة شبه منفصلة ضبابية و مجموعة منفصلة بيتا ضبابية و مجموعة منفصلة معممة منتظمة ضبابية و مجموعة منفصلة *معممة ضبابية و مجموعة منفصلة مسبقاً ضبابية مع بعض النظريات والعلاقات الأساسية ، علاوة على ذلك قمنا بتقديم ودراسة بعض المجموعات المغلقة والمفتوحة الضبابية مثل مجموعة مفتوحة معممة ضبابية ومجموعة مفتوحة الفا ضبابية و مجموعة مفتوحة بيتا ضبابية و مجموعة مفتوحة مسبقا ضبابية و ومجموعة مفتوحة *معممة ضبابية و مجموعة شبه مغلقة ضبابية و مجموعة مغلقة معممة ضبابية و مجموعة مغلقة معممة منتظمة ضبابية و مجموعة مغلقة مسبقاً ضبابية مع بعض النظريات الأساسية والعلاقات المتبادلة .