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Chapter One

Some Basic Definitions

1.1 Introduction :-

We introduce some essential fuzzy set characteristics in addition to a few fundamental definitions and theorems in this chapter.

1.2 Some Basic Definitions and Properties of Fuzzy Set.

Definition (1.2.1)[6] Let X be a nonempty set. A fuzzy set $\tilde{A}$ in X is characteristic by a member ship function. $\mu \tilde{A} : X \longrightarrow [0,1]$ and we can write this fuzzy set as : $\tilde{A} = \{ (\tilde{X}, \mu\tilde{A}\,(x)\,) : x \in X \,.\, \mu\tilde{A}\,(x) \leq 1\}$.

The collection of all fuzzy sets in X will be denoted by I^x. Where

$I^x = \{ \tilde{A} : \tilde{A}$ is fuzzy set in X } .

Definition (1.2.2) [2] The support of a fuzzy set $\tilde{A}$ is the set of all $x \in X$ such that $\mu\tilde{A}\,(x) > 0$ and is denoted by $\int(\tilde{A})$.

Definition (1.2.3)[6] A fuzzy point $\tilde{P}_x^r$ in X is a special fuzzy set with membership function defined by.

$$\tilde{P}_x^r(y) = \begin{cases} r\,, if\ x = y \\ 0\,, if\ x \neq y \end{cases}$$

Where $0 < r \leq 1$, y is the support of $\tilde{P}_x^r(x)$.

Definition (1.2.4) [3] A fuzzy set $\tilde{A}$ is said to be finite fuzzy set if $\int(\tilde{A})$ is a finite set.

Remark (1.2.5)[6]:

1) A nonempty set X is a fuzzy set with member ship $\mu x\,(x) = 1\,, \forall\, x \in X$. and X is called a crisp set.
2) A membership function $\mu\tilde{\emptyset}\,(x) = 0\ \forall\ \emptyset \in X$ is called an empty set and denoted by $\tilde{\emptyset}$.

Definition (1.2.6)[6] Let $\tilde{P}_x^r$ be a fuzzy point and $\tilde{C}$ be a fuzzy set is nonempty set $\tilde{X}$ then $\tilde{P}_x^r$ is said to be in $\tilde{C}$ or $\tilde{C}$ contains $\tilde{P}_x^r$ if $\mu\tilde{p}_x^r \leq \mu\tilde{c}$ (x) for all $x \in X$ and denoted by $X \in S\ (\tilde{c})$

Definition (1.2.7) [7] Let $\tilde{A}$ and $\tilde{B}$ be a fuzzy sets of a universal set X then.

1) $\tilde{A} \subseteq \tilde{B}$ if and only if $\mu\,\tilde{A}\,(x) \leq \mu\,\tilde{B}\,(x), \forall\, x \in X$
2) $\tilde{A} \subseteq \tilde{B}$ f and only if $\mu\,\tilde{A}\,(x) = \mu\,\tilde{B}\,(x), \forall\, x \in X$
3) $\tilde{A}^c$ is the complement of a fuzzy set $\tilde{A}$ with membership function $\mu\,\tilde{A}^c = 1 - \mu\,\widetilde{A}\,(x)$
4) $\tilde{C} = \tilde{A} \cup \tilde{B}$ if and only if $\mu\tilde{C}\,(x) = max\left\{\mu\,\tilde{A}\,(x), \mu\tilde{B}(x)\right\} \forall\, x \in X.$
5) $\widetilde{D} = \tilde{A} \cap \tilde{B}$ if and only if $\mu\widetilde{D}\,(x) = min\left\{\mu\,\tilde{A}\,(x), \mu\tilde{B}(x)\right\} \forall\, x \in X$.
6) More generally for a family of fuzzy sets, $\{\tilde{A} \propto : \propto \in \Lambda\, where\ is\ \Lambda$ is the any index set }the union $\tilde{C} = \bigcup_{\propto\in\Lambda} \tilde{A} \propto$ and the intersection $\widetilde{D} = \bigcap_{\propto\in\Lambda} \tilde{A} \propto$, and defied respectively by

$$\mu\,\tilde{c}(x) = \underset{\propto\in\Lambda}{Sup}\{\mu\,\tilde{A} \propto (x) : x \in X\}, \mu\,\widetilde{D}(x) = \underset{\propto\in\Lambda}{inf}\{\mu\,\tilde{A} \propto (x) : x \in X\}.$$

Proposition (1.2.8)[4] Let $\tilde{A}, \tilde{B}, \tilde{C}$ be fuzzy sets in X, then the following properties are satisfied .

1) **Commutatively:**
   $\max\{\mu\,\tilde{A}\,(x), \mu\,\tilde{B}\,(x)\} = \max\{\mu\,\tilde{B}\,(x), \mu\,\tilde{A}\,(x)\}\, and$
   $\min\{\mu\,\tilde{A}(x), \mu\,\tilde{B}\,(x)\} = \min\{\mu\,\tilde{B}\,(x), \mu\,\tilde{A}\,(x)\}.$
2) **Associativity** :
   $\max\{\mu\,\tilde{C}\,(x), max\left\{\mu\widetilde{A}\,(x), \mu\tilde{B}\,(x)\right\} = max\{\mu\,\tilde{A}(x), max\left\{\mu\tilde{B}(x), \mu\,\tilde{C}\,(x)\right\}.$
3) **Idempotent :**
   $\mu\tilde{A}(x) = max\left\{\mu\,\tilde{A}\,(x), \mu\,\tilde{A}(x)\right\}, \mu\,\tilde{A}(x) = min\left\{\mu\,\tilde{A}\,(x), \mu\,\tilde{A}\,(x)\right\}.$

4) **Distributive :**

$max\left\{\mu\,\tilde{A}\,(x), min\,\{\,\mu\,\tilde{B}\,(x)\}, \mu\tilde{c}(x)\right\} =$

min {max$\left\{\mu\,\tilde{A}\,(x), \mu\tilde{B}\,(x)\right\}, max\,\{\mu\,\tilde{A}\,(x), \mu\,\tilde{c}\,(x)\,\}$,

and min $\{\,\mu\,\tilde{A}\,(x), max\,\{\,\mu\,\tilde{B}\,(x), \mu\tilde{c}(x)\} =$

max $\{\,min\,\{\,\mu\,\tilde{A}\,(x), \mu\,\tilde{B}\,(x), min\left\{\mu\,\tilde{A}\,(x), \mu\,\tilde{c}\,(x)\right\}$.

5) min $\left\{\,\mu\,\tilde{A}\,(x), \mu\,\tilde{\emptyset}\,(x)\right\} = \left\{\,\mu\,\tilde{\emptyset}\,(x)\right.$, max$\left\{\,\mu\,\tilde{A}\,(x), \mu\,\tilde{x}\,(x)\right\} =$

$\mu\,\tilde{x}\,(x)$

6) **Identity**: max$\{\,\mu\,\tilde{A}\,(x), \mu\,\tilde{x}\,(x)\} = \mu\,\tilde{x}\,(x)$

7) max$\{\,\mu\,\tilde{A}\,(x)$, min$\{\,\mu\,\tilde{A}\,(x), \mu\,\tilde{B}\,(x)\} = \mu\,\tilde{A}\,(x)$

8) **De Morgan law:**

$(max\,\{\,\mu\,\tilde{A}\,(x), \mu\,\tilde{B}\,(x)\})^c = min\,\{\mu\,\tilde{A}^c\,(x), \mu\,\tilde{B}^c\,(x)\,\}$

$(min\,\{\,\mu\,\tilde{A}\,(x), \mu\,\tilde{B}\,(x)\})^c = max\,\{\mu\,\tilde{A}^c\,(x), \mu\,\tilde{B}^c\,(x)\,\}$

$(\,\mu\,\tilde{A}^c\,(x)\,)^c = \mu\,\tilde{A}\,(x)$.

Definition (1.2.9)[1] A collection $\tilde{T}$ on a fuzzy set $\tilde{A}$, such that $\tilde{T} \subseteq p\,(\tilde{A})$ is said to be a fuzzy Topology on a fuzzy set $\tilde{A}$ if it satisfied the following conditions .

1) $\tilde{A}\,, \tilde{\emptyset} \in \tilde{T}$.
2) The intersection of finite members of fuzzy sets of $\tilde{T}$ is a member of $\tilde{T}$.
3) The union of any member of $\tilde{T}$ is a member of T.

$\tilde{T}$ is called a fuzzy topology for $\tilde{X}$, and the pair $(\tilde{X}, \tilde{T})$ is an fuzzy topological spaces. Every member of $\tilde{A}$ is called $\tilde{A}$ -open fuzzy set (or simply an open fuzzy set). A fuzzy set is $\tilde{A}$ -closed if and only if its complement is $\tilde{A}$ -open.

As in general topology, the indiscrete fuzzy topology contains only $\tilde{\emptyset}$ and $\tilde{A}$, while the discrete fuzzy topology contains all fuzzy sets.

Definition (1.2.10) [10] Let $(\tilde{X}\,, \tilde{T}\,)$ be f.t.s. Let $\tilde{B}$ be a fuzzy set in $(\tilde{X}\,, \tilde{T})$, then the closure of $\tilde{B}$ (denoted by $\bar{\tilde{B}}$) and the interior of $\tilde{B}$ (denoted by $\tilde{B}^o$) are defined respectively by (i . e

$\tilde{B}^c = \cap\,\{\,\tilde{H} : \tilde{B} \subseteq \tilde{H},\ \tilde{H}^c \in \tilde{T}\,\}$, $\tilde{B}^o = \cup\,\{\,\tilde{K} : \tilde{K} \subseteq \tilde{B}, \tilde{K} \in \tilde{T}\,\}$.

Definition (1.2.11) [6] Let $(\tilde{X}, \tilde{T})$ be f.t.s then a fuzzy set $\tilde{B}$ is said to be a fuzzy neighborhood of a fuzzy point $\tilde{P}_x^r$ if there exist a f.o.s $\widetilde{H}$ in $(\tilde{X}, \tilde{T})$ such that $\mu\, \tilde{P}_x^r(x) \leq \mu\, \widetilde{H}(x) \leq \mu\, \widetilde{B}(x)$.

Example (1.2.12) : Let X = {a ,b ,c} and $\tilde{A} = \{(a, .0.8), (b, 0.7), (c, 0.4)\}$

Then $\tilde{T} = \{ \tilde{A}, \widetilde{\emptyset}, \{(a, 0.4), (b, 0.3), (c, 0.3)\} \;, (a, 0.3), (b, 0.1), (c, 0.1)\}$ is a fuzzy topological spaces on $\tilde{A}$.

Proposition (1.2.13) : Let $(\tilde{A}, \tilde{T})$ be a f.t.s , Let $\tilde{B}, \tilde{c}$ be two fuzzy sets in $(\tilde{X}, \tilde{T})$ then for all $x \in X$

1) $\mu\, \overline{\overline{\widetilde{\emptyset}}} = \mu\, \overline{\overline{\widetilde{\emptyset}}}\,(x)$.
2) $\bar{\tilde{A}}$ is a f.c.s. .
3) $if\; \widetilde{H}$ is a fuzzy closed sets , such that $\mu\tilde{B}(x) \leq \mu\widetilde{H}(x)$,when $\tilde{B}$ is a fuzzy set then $\mu\widetilde{B}\,(x) \leq \mu\, \bar{\tilde{B}}\,(x) \leq \mu\widetilde{H}\,(x)$.
4) $\tilde{B}$ is a fuzzy closed sets if and only if $\mu\tilde{B}(x) = \mu\bar{\tilde{B}}(x)$

   $\overline{\left(\max\{\mu\tilde{B}\,(x), \max \tilde{c}(x)\}\right)} = \max\left\{ \mu\bar{\tilde{B}}(x)\,, \mu\bar{\tilde{c}}(x)\right\}$.
5) $\mu\bar{\bar{\tilde{B}}} = \mu\bar{\tilde{B}}(x)$.
6) $if\; \mu\tilde{B}(x) \leq \mu\tilde{C}(x), then\; \mu\bar{\tilde{B}}(x) \leq \mu\bar{\tilde{C}}(x)$.
7) $\overline{\left(\min\{\mu\tilde{B}\,(x), \max \tilde{c}(x)\}\right)} \leq \min\left\{ \mu\bar{\tilde{B}}(x)\,, \mu\bar{\tilde{c}}(x)\right\}$.

1.3 Basic Definition and Interrelation ships

The fundamental concepts of a fuzzy T_1- space, fuzzy T_2 - space, fuzzy regular spaces, and some of its characteristics were presented in this part. are going to be study the concepts of a fuzzy T_3 - space and some of its properties. This part presents a few examples and fuzzy regular space and theorems.

Definition (1.3.1) [8] A fuzzy topological space $(\tilde{X}, \tilde{T})$ is said to be a fuzzy $T_1 -$ space if for every two distinct points $\tilde{P}_{x1}^{r1}$, $\tilde{P}_{x2}^{r2}$ there exist two fuzzy open set $\widetilde{W}_1$ and $\widetilde{W}_2$ in $\tilde{X}$, such that

$\mu^{\tilde{P}_{x1}^{r1}} \leq \mu\widetilde{W}_1$, $\mu^{\tilde{P}_{x2}^{r2}} > \mu\widetilde{W}_2$ and $\mu^{\tilde{P}_{x2}^{r2}} \leq \mu\widetilde{W}_2$, $\mu^{\tilde{P}_{x1}^{r1}} > \mu\widetilde{W}_1$.

Definition (1.3.2) [8] A fuzzy topological space $(\tilde{X}, \tilde{T})$ is said to be a fuzzy T_2-space if for every two distinct points $\tilde{P}_{x1}^{r1}$, $\tilde{P}_{x2}^{r2}$ in $\tilde{X}$ such that $\mu^{\tilde{P}_{\tilde{x}1}^{r1}} \leq \mu\widetilde{W}_1$, $\mu^{\tilde{P}_{x2}^{r2}} \leq \mu\widetilde{W}_2$, $min\{\mu\widetilde{W}_1, \mu\widetilde{W}_2\} = \emptyset$.

Definition (1.3.3) [8] A fuzzy topological space $(\widetilde{X}, \widetilde{T})$ is said to be a fuzzy regular spaces if for every a fuzzy closed set $\tilde{F}$ in $\widetilde{X}$ and a fuzzy point $\widetilde{P}_x^r$ in $\widetilde{X}$ such that $\mu\widetilde{P}_x^r > \mu\tilde{F}$ there exist two fuzzy open set $\widetilde{W}_1$ and $\widetilde{W}_2$ such that $\mu\widetilde{P}_x^r \leq \mu\widetilde{W}_1$ $\mu\tilde{F} \leq \mu\widetilde{W}_2$ and $\min\{\mu\widetilde{W}_1, \mu\widetilde{W}_2\} = \emptyset$.

Definition (1.3.4) [16] A fuzzy topological space $(\tilde{X}, \tilde{T})$ is said to be a fuzzy T_3 – space if it is a fuzzy regular and fuzzy T_1- space .

Remark (1.3.5) : Every fuzzy T_3 – spaces is a fuzzy T_2 – spaces

Theorem (1.3.6): A fuzzy topological space $(\tilde{X}, \tilde{T})$ is said to be a fuzzy regular if and only if there is a f. o. s. for every fuzzy point in $\widetilde{U}$ and every fuzzy point $\tilde{P}_x^r$ in $\widetilde{U}$ there exist a f. o. s. for every fuzzy space $\tilde{V}$ in $\tilde{X}$. Such that $\mu\tilde{P}_x^r \leq \mu\tilde{V}(x) \leq \mu\bar{\tilde{V}}(x) \leq \mu\tilde{V}(x)$.

Definition (1.3.7) [29]A fuzzy topological space $(\tilde{X}, \tilde{T})$ is said to be fuzzy Ro – space (f.Ro.s) **.** If the $\mu\tilde{P}_x^r \leq \mu\,\overline{\widetilde{U}}\,(x)$ $when\ ever\ \widetilde{U}$ is fuzzy open set in $\tilde{X}$ and $\mu\tilde{P}_x^r \leq \mu\overline{\widetilde{U}}\,(x)$.

Example (1.3.8) : Let X = {(a, 0.6) , (b, 0.6) , (c, 0.6) } and

Let $\tilde{T}$ = {$\tilde{X}$, $\widetilde{\emptyset}$ {{(a, 0.0) , (b, 0.6) , (c, 0.6)} .

Cleary is f. R∘. s But is not a fuzzy T_1 – spaces .

Theorem (1.3.9) : Every fuzzy regular spaces is a fuzzy topological space $(\tilde{X}, \tilde{T})$ then is a f. R_0. S.

Proof : Suppose that $(\tilde{X}, \tilde{T})$ is a fuzzy regular space.

We are show that $(\tilde{X}, \tilde{T})$ is a f.Ro.s. Let $\widetilde{U}$ be a f.o.s in $\tilde{X}$.

Let $\mu\tilde{P}_x^r \leq \mu\widetilde{U}\,(x)$, So $\widetilde{U}^c$ is a fuzzy closed set in $\tilde{X}$ and $\mu\tilde{P}_x^r \leq \mu\widetilde{U}\,(x)$. Since $(\tilde{X}, \tilde{T})$ is a fuzzy regular space, then there exist two fuzzy open set $\widetilde{W}_1$ and $\widetilde{W}_2$ in $\tilde{X}$ such that $\mu\widetilde{U}^c\,(x) \leq \mu\,\widetilde{W}_2$ and , min { $\mu\,\widetilde{W}_1(x), \mu\,\widetilde{W}_2$(x) } = $\widetilde{\emptyset}$. So $\mu\tilde{P}_x^r \leq \mu\,\widetilde{W}_1(x) \leq \widetilde{W}_2(x)$ such that $\widetilde{W}_x^r$ is a f.o.s in $\tilde{X}$.

$\mu\bar{\tilde{P}}_x^r \leq \mu\,\widetilde{W}_2^c\,(x)$ and Since $\mu\,\widetilde{W}_2^c \leq \mu\,\widetilde{U}$ $then$ $\mu\bar{\tilde{P}}_x^r \leq \mu\,\widetilde{U}\,(x)$and thus $(\tilde{X}, \tilde{T})$ is a f.Ro.s. .

1.4 Some Properties of Fuzzy Semi- Open Set .

The concept of fuzzy open sets and some of its characteristics are going to be studied in this part. In addition to the concept of fuzzy semi-closed sets and some of their characteristics, The fuzzy Semi- cl open set and the fuzzy Regular open set, the fuzzy Regular closed set, and finally the fuzzy Semi-cl open set are all presented in this section .

Definition (1.4.1)[17] Let $\tilde{A}$ be a fuzzy set in fuzzy topological spaces$(\tilde{X},\tilde{T})$ Then $\tilde{A}$is called a fuzzy semi-open set of X, if there exists a $\widetilde{U} \in \tilde{T}$ such that $\mu\widetilde{U}\ (x) \leq \mu\, \widetilde{A}\ (x) \leq \mu\, \overline{\overline{\widetilde{U}}}\ (x)$. A fuzzy set $\tilde{A}$ is fuzzy semi closed if and only if its complement $\tilde{A}^{c}$ is fuzzy semi-open. The class of all fuzzy semi-open (resp., fuzzy semi-closed) sets in X .

Remark (1.4.2) Every fuzzy open set is a fuzzy semi open set, however the other way need not be true as illustrated by the example following.

Example (1.4.3) Let X = {a, b, c} be a set and I = {0, 3, 5, 7, 1} of membership for fuzzy sets in $\tilde{X}$. Let $\widetilde{U}$= { (a,0.7),(b,0.0),(c,0.1)}, $\tilde{V}$= {(a,0.7),(b,0.5),(c,0.3)}, and $\widetilde{W}$= {(a,0.5),(b,0.5),(c,0.5)} be fuzzy sets on $\tilde{X}$, and $\tilde{T}$ the fuzzy topology generated by $\widetilde{U}$, $\tilde{V}$, $\widetilde{W}$. Then $\tilde{T}$ = {$\tilde{\emptyset}$, $\widetilde{U}$, $\tilde{V}$, $\widetilde{W}$, {(a,0.5), (b,0.0),(c,0.5)},{(a,0.5),(b,0.5),(c,0.3)},{(a,0.7),(b,0.0),(c,0.3)}, {(a,0.7),(b,0.0),(c,0.5)}, {(a,0.7), (b,0.5),(c,0.5)}, {(a,0.7), (b,0.5),(c,0.1)}, $\tilde{X}$}. Calculations give that fuzzy set $\widetilde{W}$ is both fuzzy semi-closed and fuzzy semi-open but $\mu\, \overline{\overline{\widetilde{W}}}^{0} = \mu\, \widetilde{W}$.

Theorem (1.4.4) A fuzzy set $\tilde{A}$ of a fuzzy topological space $(\tilde{X},\tilde{T})$ is f. s. o. s if and only if $\mu\, \widetilde{A}\ (x) \leq \mu\, \overline{\overline{\widetilde{A}}}\ (x)$.

Remark (1.4.5) :

1) The union of any fuzzy members a f. s. o. s is a f. s. o. s.
2) The intersection of two f. s. o. s need not be a f. s. o. s as the following example.

Example (1.4.6) Let X = {a , b , c }**,** T = {X , ∅ , {a, 0.4} , {b,0.6} . {(a,0.4),(b,0.6)}}

B = { (a,0.4) , (c,0.8) } , A = { (b,0.6) ,(c ,0.8)} , are f. s. o. s

But $A \cap B$ = {(c, 0.8) } is not f. s. o. s.

Proposition (1.4.7) :

1) $\mu(\widetilde{A}^{o}\ (x))^{c} = \mu\,\overline{\overline{\widetilde{A}}}^{c}(x)$.
2) $\mu\,\overline{\overline{\widetilde{A}}}^{c}(x) = \mu\,(\tilde{A}^{c}\ (x))^{o}$.

Proof. (**1**) $(\mu(\tilde{A}^{o}\ (x))^{c} = 1 - (\tilde{A}^{o}\ (x)) = 1 - \vee \{\ \widetilde{U}\ \mid \widetilde{U}\ \leq$ fuzzy open set and $\mu \leq \mu\widetilde{U} \leq \mu\,\tilde{A}\ (x) = \wedge \{\ 1 - \mu\widetilde{U} \mid \mu\widetilde{U} \leq$ fuzzy open set } and $\mu\widetilde{U} \leq \mu\,\tilde{A}\ (x) = \wedge \{$ $\mu\tilde{V}\ \mid \mu\tilde{V}^{C} \leq$ fuzzy open set } and $\mu\tilde{V} \geq \mu\,\tilde{A}^{c}$, where $\mu\,\tilde{V}\ (x) = 1 - \mu\,\tilde{A}\ (x) = \mu\,\bar{\bar{\tilde{A}}}^{c}(x)$

(2) Similar to (1).

Theorem (1.4.8)[6] A fuzzy $\tilde{B}$ of a f.t.s $(\tilde{X}, \tilde{T})$ is said to be a f.s.c.s , if and only if $\mu\left(\bar{\bar{\tilde{B}}}^{o}\ (x)\right) \leq \mu\,\tilde{B}\ (x)$.

Theorem (1.4.9)[8] If $\tilde{A}$ is f.s.o.s in and $\widetilde{U}$ is f.o.s in $\tilde{X}$ the min { $\mu\,\widetilde{U}\ (x), \mu\,\widetilde{A}\ (x)\}$ is a membership of a f. s. o. s in X.

Theorem (1.4.10) :A fuzzy set $\tilde{A}$ of a f.t.s is a f.s.o.s. if and only if there exist a f.o.s $\tilde{B}$ such that $\mu\,\widetilde{B}\ (x) \leq \mu\,\widetilde{A}\ (x) \leq \mu\,\bar{\bar{\tilde{B}}}(x)$.

Proof : first side : Let $\tilde{A}$ be a f.s.o.s , then there exist a f. o. s $\widetilde{U}\ in\ \tilde{X}$.

such that $\mu\,\widetilde{U}\,(x)\ \leq\ \mu\,\widetilde{A}\,(x)\ \leq\ \mu\,\overline{\overline{\widetilde{U}}}(x)$.

Let $\mu\,\widetilde{U}\,(x)\ \leq\ \mu\,\widetilde{A}\,(x) \leq\ \mu\,\overline{\overline{\widetilde{U}}}(x)$. Let = $\mu\,\widetilde{U}\,(x) = \mu\,\widetilde{B}\,(x)$, that $\widetilde{B}$ is a f.s.o.s and $\mu\,\tilde{B}\,(x)\,\mu\,\tilde{A}\,(x)\ \leq\ \mu\,\overline{\overline{\tilde{B}}}(x)$.

Second side : Let $\tilde{B}$ be a f.s.o.s , such that $\mu\,\tilde{B}\,(x) \leq\ \mu\,\tilde{A}\,(x) \leq\ \mu\,\overline{\overline{\tilde{B}}}(x)$ …… (1) **.** Since$\widetilde{B}$ is a f.s.o.s, then there exist a f.o.s $\widetilde{U}$ such that

$\mu\,\widetilde{U}\,(x) \leq\ \mu\,\tilde{B}\,(x) \leq\ \mu\,\overline{\overline{\widetilde{U}}}(x)$ …… (2) . From (1) and (2) we get

$\mu\,\widetilde{U}\,(x) \leq\ \mu\,\bar{\bar{\tilde{B}}}(x) \leq\ \mu\,\overline{\overline{\widetilde{U}}}(x)$. So $\mu\,\widetilde{U}\,(x) \leq\ \mu\,\tilde{A}\,(x) \leq\ \mu\,\overline{\overline{\widetilde{U}}}$.

There four $\tilde{A}$ is a f. s. o. s. , of $\tilde{X}$.

Theorem (1.4.11) If $(\tilde{X}, \tilde{T})$ is a f. t. s and let $\widetilde{Y}$ a fuzzy subspace of $\tilde{X}$ such that $\mu\,\tilde{A}\,(x) \leq\ \mu\,\tilde{Y}\,(x)\ \leq\ \mu\,\tilde{X}\,(x)$ and $\tilde{A}$ is a f.s.o.s in $\tilde{X}$, then $\tilde{A}$ is a f.s.o.s in $\widetilde{Y}$.

Theorem (1.4.12) Let $\widetilde{U}$ be a f.o.s in $\tilde{X}$ and $\tilde{A}$ be a f.s.o.s in $\tilde{X}$, then

min $\{\mu\,\widetilde{U}\,(x)\,, \mu\,\widetilde{A}\,(x)\}$ is a f.s.o.s in $\widetilde{U}$.

Remark (1.4.13) [6] The intersection of a f. s. c. s is a f. s. c. s.

Definition (1.4.14) [32] Let $(\tilde{X}, \tilde{T})$ be a f.t.s and $\tilde{A}$ be a fuzzy set of $\tilde{X}$, then the intersection of all f.s.c.s which contain $\tilde{A}$ is called (semi – closure of $\tilde{A}$) and denoted by($f\ ((\bar{\bar{\tilde{A}}})^{s})^{c}$) .

Example (1.4.15) Let $\tilde{X}$ = { (a, 0.6) , (b , 0.6) , (c , 0.6) }

$\tilde{T} = \{ \tilde{X}, \tilde{\emptyset}$, { (a, 0.6) , (b , 0.0) , (c , 0.0) }, { (a, 0.6) , (b , 0.6) , (c , 0.0) }} . Take $\tilde{A}$ = { (a, 0.0) , (b , 0.6) , (c , 0.6) } . So $\tilde{A}$ is a f. semi – closure.

Remark (1.4.16) Let $(\tilde{X}, \tilde{T})$ be a fuzzy topological spaces and $\mu\,\tilde{A}\,(x) \leq \mu\,\tilde{B}\,(x) \leq \mu\,\tilde{X}\,(x)$ then .

1) $\mu\,\tilde{A}\,(x) \leq \mu\, f\ \overline{(\tilde{A})^s} \leq \mu\,\bar{\tilde{A}}\,(x)$.

2) $\mu\, f\ \overline{(\tilde{A})^s} \leq \mu\, f\ \overline{((\tilde{B})s\,)}$.

3) $\mu\,\overline{\overline{(\,(\tilde{A})^s)^s}} = \mu.F\ \overline{(\bar{\tilde{A}})^s}$.

4) A fuzzy set $\tilde{A}$ is a f,s.c.s if and only if $\mu\,\tilde{A}\,(x) = \mu\, f\,(\bar{\tilde{A}})^s$.

5) $f\,(\bar{\tilde{A}})^s$ is the smallest a f.s.c.s contains $\tilde{A}$.

6) $max\,\{\mu\, f\ \overline{(\tilde{A})}^s . \mu\, f\ \overline{(\tilde{B})^s\}} \leq f.\max\ \overline{(\,\mu\,\tilde{A}\,(x), \mu\,\tilde{A}\,(x))}^s$.

7) $f.\overline{\{\mu\left(\tilde{A}\,(x),\ \overline{(\tilde{A})}^s\right)}, min\,\{f.\mu\,\overline{(\tilde{A}}\,)^s, f.\mu\,\overline{\left(\tilde{B}\,)^s\right\}}$.

Theorem (1.4.17) If $\tilde{A}$ is a f.c.s of $(\tilde{X}, \tilde{T})$ and $\mu\,\tilde{A}^o\,(x) \leq \mu\,\tilde{B}\,(x) \leq \mu\,\tilde{A}\,(x), then\ \tilde{B}$ is a f.c.s.

Proof:- Let $\tilde{A}$ is a f.s.o.s , since $\mu\,\tilde{A}^o\,(x) \leq \mu\,\tilde{B}\,(x) \leq \mu\,\tilde{A}\,(x)$ (by hypothesis) . $\mu\,\tilde{A}^c\,(x) \leq \mu\,\tilde{B}^c\,(x) \leq \mu(\,\tilde{A}^o)^c$ **.** But $(\,\tilde{A}^o\,(x))^c = \mu\,(\,\bar{\tilde{A}}^c)$ by (1.4.7) . Then $\mu\,\tilde{A}^c\,(x) \leq \mu\tilde{B}^c(x) \leq \mu\,\bar{\tilde{A}}^c$ **,** and by (theorem 1.4.10) , $\tilde{B}^c$ is a f. o. s **,** So $(\tilde{B}^c\,)^c$ is a f.c.s , which means $\tilde{B}$ is a f.c.s.

Remark (1.4.18) :

A fuzzy set $\tilde{B}$ of a f.t.s $(\tilde{X}, \tilde{T})$ is a f.s.c.s . if and only if there exist a f.c.s, $\tilde{F}$such that $\mu\, \tilde{F}^c\,(x) \leq \mu\, \tilde{B}\,(x) \leq \mu\, \tilde{F}\,(x)$.

Definition(1.4.19)[32]Let$(\tilde{X}, \tilde{T})$ be a f.t.s and let $\mu\, \tilde{A}\,(x) \leq \mu\, \tilde{B}\,(x), \mu\, \tilde{P}_x^r \leq \tilde{X}\,(x)$, then $\tilde{P}_x^r$ is said to be (semi – interior point) if and only if there exist a f.o.s, $\tilde{U}$ such that $\mu\, \tilde{P}_x^r\, \mu\, \tilde{U}\,(x) \leq \mu\, \tilde{A}\,(x)$.

Remark (1.4.20): Let $(\tilde{X}, \tilde{T})$be a f. t. s and $\mu\, \tilde{A}\,(x) \leq \mu\, \tilde{B}\,(x) \leq \mu\, \tilde{X}\,(x)$ then .

1) $\mu\, \tilde{A}^o\,(x) \leq \mu\, \overline{(\tilde{A})}^s \leq \mu\, \tilde{A}\,(x)$.
2) $\mu\, \tilde{A}^o\,(x) \leq \mu\, \tilde{B}^o$.
3) $\overline{(\tilde{\mu})}^{os} = \mu\, \tilde{A}^{os}$.
4) $\tilde{A}$ is a f.o.s if and only if $\mu\, \tilde{A}^{os} = \mu\, \tilde{A}(x)$.

Theorem (1.4.21) Let $(\tilde{x}, \tilde{T})$be a f.t.s , $\mu\, \tilde{A}\,(x) \leq \mu\, \tilde{X}\,(x)$,then $\mu\, \tilde{P}_x^r \leq \mu f\, (\bar{\tilde{A}})^s$ if and only if every a f.s.o.s $\tilde{U}$ which contains $\tilde{P}_x^r$, $\tilde{U}$ contains at least one fuzzy point of $\tilde{A}$ that $Min\,\{\mu\, \tilde{A}\,(x), \mu\, \tilde{U}\,(x)\} \neq 0$.

Proof : First side Let $\mu\, \tilde{P}_x^r \leq \mu f(\bar{\tilde{A}})^s$, and let $\tilde{U}$ be a f. s. o. s such that $\mu\, \tilde{P}_x^r \leq \mu\, \tilde{U}(x)$ and **,** $min\,\{\mu\, \tilde{A}\,(x), \mu\, \tilde{U}\,(x)\} = 0$, then $\tilde{U}^c$ is a f.s.c.s and $\mu\, \tilde{P}_x^r > \mu \tilde{U}^c(x)$. Since $f(\bar{\tilde{A}})^s$ is the intersection of all f.s.c.s , which contain $\tilde{A}$ and , $\mu\, \tilde{P}_x^r \leq \mu f\,(\bar{\tilde{A}})^s$the $\mu\, \tilde{P}_x^r < \mu\, \tilde{U}^c$,which is contradiction so $min\,\{\mu\, \tilde{A}\,(x), \mu\, \tilde{U}\,(x)\} \neq 0$.

Second side: Let $\tilde{U}$ be a f.s.o.s and $\mu\, \tilde{P}_x^r \leq \mu\, \tilde{U}(x)$ and $min\,\{\mu\, \tilde{A}\,(x), \mu\, \tilde{U}\,(x)\} \neq 0$. To show that $\mu\, \tilde{P}_x^r \leq \mu f\,(\bar{\tilde{A}})^s$, Let $\mu\, \tilde{P}_x^r > \mu f(\bar{\tilde{A}})^s$. So the exist a f. c. s , $\tilde{V}$ contains $\tilde{A}$ and $\mu \tilde{P}_x^r > \mu \tilde{V}\,(x)$, $\tilde{V}^c$a f.s.o.s **.**

And $\mu \tilde{P}_x^r < \mu \tilde{V}^c(x)$.So $min\{\mu \tilde{V}^c(x), \mu\tilde{A}(x)\} = 0$. Which contraction , therefore $\mu \tilde{P}_x^r \leq \mu f(\bar{\tilde{A}})^s$.

Definition (1.4.22) [17] A fuzzy set $\tilde{B}$ of a fuzzy topological spaces$(\tilde{X}, \tilde{T})$ is said to be a fuzzy regular open set (f.r.o.s.) if $\mu \bar{\tilde{B}}(x) = \mu\bar{\tilde{B}}^0$(x) .

Remark (1.4.23) Every a f. r. o. s. is a f. o. s. and the converse need not be true as the following Example.

Example (1.4.24) Let $\tilde{X} = \{(a, 0.7), (b, 0.6), (c, 0.4)\}$ and

Let $\tilde{T} = \{\tilde{\emptyset}, \tilde{X}, \{(a, 0.3), (b, 0.0), (c, 0.0)\}$

Bea f. t. s de find on $\tilde{X}$.Take $\tilde{A} = \{(a, 0.3), (b, 0.0), (c, 0.0)\}$ clearly

$\tilde{A}$ is a f.o.s but is not a f.r.o.s.

Definition (1.4.25)[28] A fuzzy set $\tilde{C}$ of a fuzzy topological spaces$(\tilde{X}, \tilde{T})$ is said to be a fuzzy regular closed set (f.r.c.s.) if $\mu \tilde{C}(x) = \mu\bar{\tilde{C}}^0$(x) .

Remark (1.4.26) Every f. r. c. s. is a f. c. s and the converse need not be true as the following example :-

Example (1.4.27) In the example (1.4.24) take

$\tilde{D} = \{(a, 0.4), (b, 0.6), (c, 0.4)\}$ clearly $\tilde{D}$ is a f. c. s But not f. r. c. s. .

Theorem (1.4.28) Let $(\tilde{X}, \tilde{T})$ be a f.t.s and let:-

1) $\tilde{U}$ be a f.o.s in $\tilde{X}$, then $\overline{(\bar{\tilde{U}})^o}$ is a f.r.o.s.
2) $\tilde{F}$ is a f.o.s in $\tilde{X}$, then $\overline{(\bar{\tilde{F}})^o}$ is a f.r.o.s.

Proof (1) To show $\bar{\tilde{U}}^0$ is a Fuzzy regular open set . we have to show

$\mu \overline{(\bar{\tilde{U}})^o} = \mu \overline{((\bar{C})^o}\,)^o$ .Now since $\mu \bar{\tilde{U}}^0 \leq \mu\bar{\tilde{U}}$ (x) then $\mu \overline{((\bar{C})^o}\,)^o \leq \mu\bar{\bar{\tilde{U}}}$(x) = $\mu\bar{\tilde{U}}$ (x) . So $\mu \overline{((\bar{C})^o}\,)^o \leq \mu \overline{(\bar{\tilde{U}})^o}$ …. (1)

الخلاصة

قمنا في هذا العمل بدراسة بعض انواع الفضاءات المنتظمة الضبابية بما في ذلك الفضاءات المنتظمة الضبابية شبه المغلقة و الفضاءات المنتظمة الضبابية الشبه المفتوحة و الفضاءات المنتظمة الضبابية والفضاءات الشبه المنتظمة و الشبه الغامضة ثم تعميمها و درسنا بعض النظريات الحفظ التي تخص هذه الفضاءات الضبابية وبعض الخصائص الضبابية الوراثية الغامضة التي ترتبط بهذه الفضاءات الغامضة وتوصلنا الى بعض النتائج المهمة :

اذا كان الفضاء الضبابي $(\tilde{X}, \tilde{T})$ عبارة عن فضاءات منتظمة غامضة و فضاءات شبه متماثلة غامضة فأن الفضاء الضبابي $(\tilde{X}, \tilde{T})$ هي فضاءات شبه ضبابية غامضة .

الفضاء الضبابي $(\tilde{X}, \tilde{T})$ هي فضاءات شبه غامضة و فضاءات منتظمة معممة اذا وفقط اذا $(\tilde{X}, \tilde{T})$ هي عبارة فضاءات شبه منتظمة وشبه ضبابي .

الفضاء الضبابي $(\tilde{X}, \tilde{T})$ هي فضاءات عادية معممة غامضة اذا كان الفضاء $(\tilde{X}, \tilde{T})$ هي عبارة عن فضاءات عادية و غامضة و فضاءات

الفضاء الضبابي $(\tilde{X}, \tilde{T})$ هي فضاءات عادية معممة غامضة اذا كانت توجد مجموعة مفتوحة $\mu\tilde{P}_x^r \leq \tilde{U}$ in $\tilde{X}$ $\mu\tilde{U}(x)$ حيث لكل مجموعة مفتوحة غامضة اما اذا كانت هناك مجموعة مفتوحة معممة غامضة بحيث ان لكل .

$$\mu\tilde{P}_x^r \leq \mu\tilde{V}(x) \leq \mu\,((\bar{\tilde{V}})^g(x) \leq \mu\tilde{U}(x).$$

ليكن الفضاء الضبابي $(\tilde{X}, \tilde{T})$ هو الفضاء الطوبولوجية الغامضة فأن العبارات التالية متكافئة .

i. الفضاء $(\tilde{X}, \tilde{T})$ هي فضاءات شبه منتظمة غامضة او فضاءات منتظمة شبه مفتوحة غامضة .

ii. بالنسبة للنقطة الغامضة $\tilde{P}_x^r$ in $\tilde{X}$ و لكل الفضاءات شبه المفتوحة الغامضة $\tilde{A}$ حيث تحتوي على النقطة الغامضة توجد فضاءات شبه مفتوحة غامضة $\tilde{B}$