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Existence and Uniqueness of Solution for COVID-19 Model as an Application of Fractional Differential Equation

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Received: 15 November 2022 **Accepted: 25 January 2023**

DOI: https://doi.org/10.24237/ASJ.02.01.711B

Abstract

In this work, we study an impulsive mathematical model proposed by Ndaïrou et al [1]to describe the dynamics of COVID-19 model. To improve our understanding of this biological phenomenon that has emerged in China in recent years and then spread throughout the world. Has resulted in the death of millions of people. We the existence and uniqueness of the model for show that the model has the unique solution by using and proving some theorems. These theories help us to establish the patient's condition and its recovery in a hurry.

Keywords: existence solution, uniqueness solution stability, COVID-19 model, equilibrium point.

اثبات وجود الحل و تفرده لمنظومة كوفيد-١٩ باعتباره تطبيق المعادلة التفاضلية الكسرية

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الخلاصة

في هذا العمل ، ندرس نموذجًا رياضيًا اندفاعيًا اقترحه Ndaïrou et al [1] لوصفه ديناميكيات نموذج COVID-19. لفهم أفضل لهذه الظاهرة البيولوجية التي ظهرت في السنوات الأخيرة في الصين ثم انتشرت في جميع أنحاء العالم والتي ادت إلى وفاة الملايين من الناس. حيث تحققنا من امكانية وجود النموذج وتفرده و توصلنا ان لهذا النموذج حل وحيد

باالعتماد على بعض النظريات التي قمنا برهناها في سياق البحث. حيث ساعدتنا هذه النظريات على تشخيص حالة المريض و العمل على شفائه بسرعة.

كلمات مفتاحية : وجوديه و وحدانيه الحل، الاستقرارية، كوفيد_١٩، نقطة الحرجة.

Introduction

In this work present the coronavirus COVID-19 mathematical models. The spread of the COVID-19 pandemic is modelled using a fractional compartmental mathematical model. The transmissibility of super-spreaders has received special attention. Coronavirus disease 2019 (COVID-19), the outbreak due to severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), has taken on pandemic proportions in 2020affecting more than 1.5 million individuals in almost all countries [2], To combat the COVID-19 pandemic, an integrated science and multidisciplinary approach is required [3]. Mathematical and epidemiological simulations, in particular, are essential for forecasting, anticipating, and managing current and future outbreaks. The sickness was originally discovered in Wuhan, and China’s capital, in December 2019 and has since spread around the world, resulting in the continuing 2020 pandemics outbreak. Because of thousands of confirmed infections and thousands of deaths around the world, the COVID-19 pandemic is considered the most serious global threat. Report confirmed 663640386 cumulative cases with 6713093 deaths. According to a World Health Organization report dated 20 October 2022, [4] the numbers had risen to 1,353,361 verified cumulative cases with 79,235 deaths at the time of this revision [1, 5]. As a result, fractional derivatives, which have been widely employed to generate models of infectious diseases since they account for the memory effect, appear to be acceptable. Estimates of COVID-19 infected persons, generated a priori using mathematical models, have aided in predicting the number of required beds for both hospitalized individuals and, in particular, intensive care units. In this research, we study the existence and uniqueness of solution for fractional differential equations with Caputo fractional derivative of order α. Which is a continuous function.

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1. The Proposed COVID-19 Fractional Model

Research on fractional differential equations is ongoing, and it is sometimes permissible to acknowledge the progressions' history. The fractional proposed model takes the form

$$\frac{\partial^\alpha x_1}{\partial t^\alpha} = -\beta\frac{x_3}{N}x_1 - \iota\beta\frac{x_6}{N}x_1 - \acute{\beta}\frac{x_4}{N}x_1..$$
$$\frac{\partial^\alpha x_2}{\partial t^\alpha} = \beta\frac{x_3}{N}x_1 + \iota\beta\frac{x_6}{N}x_1 + \acute{\beta}\frac{x_4}{N}x_1 - \kappa x_2.$$
$$\frac{\partial^\alpha x_3}{\partial t^\alpha} = \kappa\rho_1 x_2 - (\gamma_a + \gamma_i + \delta_i)x_3.$$
$$\frac{\partial^\alpha x_4}{\partial t^\alpha} = \kappa\rho_2 x_2 - (\gamma_a + \gamma_i + \delta_p)x_4. \qquad (1.1)$$
$$\frac{\partial^\alpha x_5}{\partial t^\alpha} = \kappa(1 - \rho_1 - \rho_2)x_2.$$
$$\frac{\partial^\alpha x_6}{\partial t^\alpha} = \gamma_a(x_3 + x_4) - (\gamma_r + \delta_h)x_6.$$
$$\frac{\partial^\alpha x_7}{\partial t^\alpha} = \gamma_i(x_3 + x_4) + \gamma_r x_6.$$
$$\frac{\partial^\alpha x_8}{\partial t^\alpha} = \delta_i x_3 + \delta_p x_4 + \delta_h x_6.$$

Where x_1 is susceptible individuals, x_2 represent exposed individuals, (x_3) represent symptomatic and infectious individuals, x_4 is super-spreaders individuals, x_5 is infectious but asymptomatic individuals, x_6 is hospitalized individuals, recovery individuals (x_7), and dead individuals (x_8) or fatality class. And $x_i(0) > 0$ for $i = 1,2, \ldots ,8$

Consequently, we have the following parameters:

Table 1: parameters of the models

β	COMMUNICATION FROM HUMAN TO HUMAN
$\acute{\beta}$	High communication quantity due to super-spreaders
l	The relative transmissibility of hospitalized patients
κ	Different leaf the visible class by becoming communicable
ρ_1	Evolution from exposed class $x2$ to symptomatic communicable class $x3$
ρ_2	Low rate at which visible people become super-speared
$1 - \rho_1 - \rho_2$	Progress from visible to asymptomatic class
γ_a	Average rate at which asymptomatic and super-speareds people become hospitalized
γ_i	The repossession rate without being hospitalized
γ_r	Repossession rate of hospitalized patients
δ_i	Illness caused by death rates due to infected individuals
δ_p	Illness caused by death rates due to super-spreads individuals
δ_h	Illness caused by death rates due to hospitalized individuals.

2. Existence and uniqueness:

In this section, we study the existence and uniqueness of model (1.1)

2.1 Existence

The following condition must be satisfied for the solution to exist in the expedient space of function [5]

(C1) let $f(t, x_1, x_2, \dots, x_8) \in C((0,T] \times \mathbb{R}_1 \times, .. \times \mathbb{R}_8)$ and

$x^{\alpha} f(t, x_1, x_2, \dots, x_8) \in C([0,T] \times \mathbb{R}_1 \times, .. \times \mathbb{R}_8)$, where $C(x_i)$ represents the class of continuous functions defined on $x_i, i = 1,2, \dots, 8$.

Set the initial values at $x_1(0) \geq 0, x_2(0) \geq 0, \dots, x_8(0) \geq 0$. This Existence if $f_i(t, X(t))$ is continues for fractional Cupto method.

$$\dot{x}_1{}^{\alpha} = -\beta \frac{x}{N} x_1 - \iota\beta \frac{x_6}{N} x_1 - \acute{\beta} \frac{x_4}{N} x_1$$

Let the function

$$f_1(t, X(t)) = -\beta \frac{x_3}{N} x_1 - \iota\beta \frac{x_6}{N} x_1 - \acute{\beta} \frac{x_4}{N} x_1.$$

So that, show that $f_1$is Continuous on the reign to and find the derivatives of fractional. We use the fractional derivative in
the sense Cupto method.

$$D^{\alpha} f_1(t, X(t)) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} f'(s) ds$$

and $\alpha = \frac{1}{2}$,

$$f_1{}'(x_1) = -\beta \frac{x_3}{N} - \iota\beta \frac{x_6}{N} - \acute{\beta} \frac{x_4}{N}$$

$$D^{\alpha} f_1(t, X(t)) = \frac{1}{\Gamma\left(\frac{1}{2}\right)} \int_0^t (t-s)^{-\frac{1}{2}} (-\beta \frac{x_3}{N} - \iota\beta \frac{x_6}{N} - \acute{\beta} \frac{x_4}{N}) ds$$

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$$=\frac{(-\beta\frac{x_3}{N}-\iota\beta\frac{x_6}{N}-\acute{\beta}\frac{x_4}{N})}{\sqrt{\pi}}2t^{\frac{1}{2}}$$

It follows that, f_1 is Continuous

And

$$\dot{x}_2{}^{\alpha}=\beta\frac{x_3}{N}x_1+\iota\beta\frac{x_6}{N}x_1+\acute{\beta}\frac{x}{N}x_1-\kappa x_2$$

$$D^{\alpha}f_i(t,X(t))=\frac{1}{\Gamma(1-\alpha)}\int_0^t(t-s)^{-\alpha}f'(s)ds$$

$$f_2(t,X(t))=\beta\frac{x_3}{N}x_1+\iota\beta\frac{x_6}{N}x_1+\acute{\beta}\frac{x}{N}x_1-\kappa x_2$$

$$f_2{}'(x_2)=-\kappa$$

$$D^{\alpha}f_2(t,X(t))=\frac{1}{\Gamma\left(\frac{1}{2}\right)}\int_0^t(t-s)^{-\frac{1}{2}}(-\kappa)ds$$

$$=\frac{1}{\sqrt{\pi}}(-\kappa)2t^{\frac{1}{2}}$$

then, f_2 is Continuous.

Similarly using the same way to show that $f_3,\dots,f_7$ and f_8 are Continuous.

Theorem. 2.1.[6] **(Schaefer's critical-point theorem**). Let X be a real Banach space $B\subset X$ Non-empty closed bounded and convex, $M:B\to B$ compact. Then, M has a critical point.

Lemma 2.2.1: Under the condition (C1), if $\omega_i\in C^{\alpha}[0,T]$ is a solution of the models (1.1), then $\omega_i\in C^{\alpha}[0,T]$ is solution of the following integral equation

$$\omega_i(t)=\frac{b_i}{\Gamma(\alpha)}t^{\alpha}+\frac{1}{\Gamma(\alpha)}\int_0^t\frac{f_i(t,X(t))}{(t-x_i)^{1-\alpha}}dx_i \qquad (2.1)$$

and, vice versa.

In order to prove this lemma first to find b_i by a critical point $E_0=(a,0,0,0,b,0,c,d)$ such that $b_i=D^{\alpha}f_i(X(t))$. Then b_i in the critical point this implies that

$$b_1=D^{\alpha}f_1(t,X(t))=\frac{1}{\Gamma(1-\alpha)}\int_o^x(t-x_1)^{-\alpha}f_1{}'(s)ds$$

then in the $E_0=(a,0,0,0,b,0,c,d)$ and $f_1{}'(x_1)=(-\beta\frac{x_3}{N}-I\beta\frac{x_6}{N}-\acute{\beta}\frac{x_4}{N})$

$$b_1 = D^{\alpha} f_1(X(t)) = \frac{(-\beta\frac{x_3}{N} - I\beta\frac{x_6}{N} - \acute{\beta}\frac{x_4}{N})}{\Gamma\left(1-\frac{1}{2}\right)} \int_o^x (t-x_1)^{-1/2}\, dx_1$$

$$= \frac{(-\beta\frac{x_3}{N} - I\beta\frac{x_6}{N} - \acute{\beta}\frac{x_4}{N})}{\sqrt{\pi}} 2t^{\frac{1}{2}}$$

at the E_0 then $b_1 = 0$.

Similarly using the same way to calculate $b_2, \dots, b_8$ we implies that $b_2 = -\frac{2t^{\frac{1}{2}}}{\sqrt{\pi}}\kappa, b_3 = -\frac{(\gamma_a+\gamma_i+\delta_i)}{\sqrt{\pi}} 2t^{\frac{1}{2}}, b_4 = -\frac{(\gamma_a+\gamma_i+\delta_p)}{\sqrt{\pi}} 2t^{\frac{1}{2}}, b_5 = b_7 = b_8 = \frac{1}{\sqrt{\pi}}, b_6 = -\frac{(\gamma_r+\delta_h)}{\sqrt{\pi}} 2t^{\frac{1}{2}}$.

$$B = \begin{pmatrix} 0 \\ -\frac{2t^{\frac{1}{2}}}{\sqrt{\pi}}\kappa \\ -\frac{(\gamma_a+\gamma_i+\delta_i)}{\sqrt{\pi}} 2t^{\frac{1}{2}} \\ -\frac{(\gamma_a+\gamma_i+\delta_p)}{\sqrt{\pi}} 2t^{\frac{1}{2}} \\ \frac{1}{\sqrt{\pi}} \\ -\frac{(\gamma_r+\delta_h)}{\sqrt{\pi}} 2t^{\frac{1}{2}} \\ \frac{1}{\sqrt{\pi}} \\ \frac{1}{\sqrt{\pi}} \end{pmatrix}$$

Proof. Suppose that $\omega_i \in C^{\alpha}[0,T]$ is a solution of the models. For a solution of the existence the first to find b_i such that $b_i = D^{\alpha} f_i(x)$ at the critical point we get for

$$b_1 = 0$$

so that to find $\omega_i(t)$ and $\alpha = \frac{1}{2}$,

$$\omega_i(t) = \frac{b_i}{\Gamma(\alpha)} t^{\alpha} + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f_i(t, X(t))}{(t-x_i)^{1-\alpha}} dx_i$$

$$\omega_1(t) = \frac{b_1}{\Gamma(\alpha)} t^{\alpha} + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f_1(t, X(t))}{(t-x_1)^{1-\alpha}} dx_1$$

$$\omega_1(t)=\frac{0}{\Gamma(\frac{1}{2})}t^{\frac{1}{2}}+\frac{1}{\Gamma(\frac{1}{2})}\int_0^t\frac{(-\beta\frac{x_3}{N}x_1-\iota\beta\frac{x_6}{N}x_1-\acute{\beta}\frac{x_4}{N}x_1)}{(t-x_1)^{\frac{1}{2}}}dx_1$$

$$=0t^{\frac{1}{2}}+\frac{(-\beta\frac{x_3}{N}-\iota\beta\frac{x_6}{N}-\acute{\beta}\frac{x_4}{N})}{\sqrt{\pi}}\int_0^t x_1(t-x_1)^{-\frac{1}{2}}dx_1$$

in the equation let $\beta\frac{x_3}{N}+\iota\beta\frac{x_6}{N}+\acute{\beta}\frac{x_4}{N}=M$ such that

$$=0+\frac{M}{\sqrt{\pi}}\int_0^t -x_1(t-x_1)^{-\frac{1}{2}}dx_1$$

$$=\frac{M}{\sqrt{\pi}}\int_0^t -x_1(t-x_1)^{-\frac{1}{2}}dx_1$$

$$=-\frac{4M}{3\sqrt{\pi}}(t)^{\frac{3}{2}}$$

Therefore, let $b_2=-\frac{2t^{\frac{1}{2}}}{\sqrt{\pi}}\kappa$ and $f_2(t,X(t))=\left(\beta\frac{x_3}{N}x_1+\iota\beta\frac{X_6}{N}x_1+\acute{\beta}\frac{X_4}{N}x_1-\kappa x_2\right)$, such that

$$\omega_i(t)=\frac{b_i}{\Gamma(\alpha)}t^{\alpha}+\frac{1}{\Gamma(\alpha)}\int_0^t\frac{f_i(t,X(t))}{(t-x_2)^{1-\alpha}}dx_2$$

$$\omega_2(t)=\frac{b_2}{\Gamma(\alpha)}t^{\alpha}+\frac{1}{\Gamma(\alpha)}\int_0^t\frac{f_2(t,X(t))}{(t-x_2)^{1-\alpha}}dx_2$$

$$\omega_2(t)=\frac{-\frac{\kappa}{\sqrt{\pi}}2t^{\frac{1}{2}}}{\Gamma(\frac{1}{2})}t^{\frac{1}{2}}+\frac{1}{\Gamma(\frac{1}{2})}\int_0^t\frac{(\beta\frac{x_3}{N}x_1+\iota\beta\frac{x_6}{N}x_1+\acute{\beta}\frac{x_4}{N}x_1-\kappa x_2)}{(t-x_2)^{\frac{1}{2}}}dx_2$$

$$=\frac{-\frac{\kappa}{\sqrt{\pi}}2t^{\frac{1}{2}}}{\sqrt{\pi}}t^{\frac{1}{2}}+\frac{(\beta\frac{x_3}{N}x_1+\iota\beta\frac{x_6}{N}x_1+\acute{\beta}\frac{x_4}{N}x_1)}{\sqrt{\pi}}\int_0^t(t-x_2)^{-\frac{1}{2}}dx_2$$

$$+\frac{\kappa}{\sqrt{\pi}}\int_0^t -x_2(t-x_2)^{-\frac{1}{2}}dx_2$$

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then in the equation let $J = (\beta \frac{x_3}{N} x_1 + \iota\beta \frac{x_6}{N} x_1 + \acute{\beta} \frac{x_4}{N} x_1)$ such that

$$-\frac{2t\kappa}{\pi} + \frac{J}{\sqrt{\pi}} \int_0^t (t - x_2)^{-\frac{1}{2}} dx_2 + \frac{\kappa}{\sqrt{\pi}} \int_0^t -x_2(t - x_2)^{-\frac{1}{2}} dx_2$$
$$= -\frac{2t\kappa}{\pi} + \frac{J}{\sqrt{\pi}} 2t^{\frac{1}{2}} + \frac{\kappa}{\sqrt{\pi}} 2 \int_0^t (t - x_2)^{\frac{1}{2}} dx_2$$
$$= -\frac{2t\kappa}{\pi} + \frac{J}{\sqrt{\pi}} 2t^{\frac{1}{2}} - \frac{4}{3} \frac{\kappa}{\sqrt{\pi}} t^{\frac{3}{2}}.$$

Then similarly using same ω_1 to find $\omega_3, \dots, \omega_8$.

In this lemma $\Omega = \begin{pmatrix} -\frac{4M}{3\sqrt{\pi}}(t)^{\frac{3}{2}} \\ -\frac{2t\kappa}{\pi} + \frac{J}{\sqrt{\pi}} 2t^{\frac{1}{2}} - \frac{4}{3}\frac{\kappa}{\sqrt{\pi}} t^{\frac{3}{2}} \\ \frac{-(\gamma_a+\gamma_i+\delta_i)2t}{\pi} + \frac{2t^{\frac{1}{2}}}{\sqrt{\pi}} \kappa\rho_1 x_2 - \frac{4}{3}\frac{(\gamma_a+\gamma_i+\delta_i)}{\sqrt{\pi}} t^{\frac{3}{2}} \\ \frac{-(\gamma_a+\gamma_i+\delta_p)2t}{\pi} + \frac{2t^{\frac{1}{2}}}{\sqrt{\pi}} \kappa\rho_2 x_2 - \frac{4}{3}\frac{(\gamma_a+\gamma_i+\delta_p)}{\sqrt{\pi}} t^{\frac{3}{2}} \\ \left(\frac{1}{\pi} + \frac{2(\kappa(1-\rho_1-\rho_2)x_2)}{\sqrt{\pi}}\right) t^{\frac{1}{2}} \\ \frac{-2(\gamma_r+\delta_h)t}{\pi} + \frac{\gamma_a(x_3+x_4)}{\sqrt{\pi}}\left(2t^{\frac{1}{2}}\right) + \frac{4}{3}\frac{(\gamma_r+\delta_h)}{\sqrt{\pi}} t^{\frac{3}{2}} \\ \left(\frac{1}{\pi} + \frac{2(\gamma_i(x_3+x_4)-\gamma_r x_6)}{\sqrt{\pi}}\right) t^{\frac{1}{2}} \\ \left(\frac{1}{\pi} + \frac{2(\delta_i x_3+\delta_p x_4+\delta_h x_6.)}{\sqrt{\pi}}\right) t^{\frac{1}{2}} \end{pmatrix}$ is a solution in the space $C^{\alpha}[0, T]$.

Theorem 2.2. (Existence). If (C1) is satisfied, and t $\left|t^{\alpha} f_i(t, u_1, u_2, \dots, u_8)\right| \leq m$ for All real positive numbers n, m , and$(t, u_1, u_2, \dots, u_8) \in I$ Equipped with $n_1 + n_2 + \dots + n_8 \leq n$, then model (1.1) states at least one solution in $C^{\alpha}[0, T]$, where

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$$T_0=\begin{cases} T & if \quad T<\dfrac{n}{C(b_i,\alpha,m)} \\ \dfrac{n}{C(b_i,\alpha,m)} & if \quad T\geq \dfrac{n}{C(b_i,\alpha,m)}\geq 0 \\ \dfrac{n}{C(b_i,\alpha,m)} & if \quad T\geq \dfrac{n}{C(b_i,\alpha,m)},\quad 0\geq \dfrac{n}{C(b_i,\alpha,m)} \ and\ 0<\alpha\leq 0.5 \\ \dfrac{n}{C(b_i,\alpha,m)} & if \quad T\geq \dfrac{n}{C(b_i,\alpha,m)},\quad 0\geq \dfrac{n}{C(b_i,\alpha,m)},\ and\ 0.5\leq \alpha<1 \end{cases}$$

And

$$C(b_i,\alpha,m)=\left[\frac{|b_i|}{\Gamma(\alpha)}+m\left(\frac{1+\Gamma(2-\alpha)}{1-\alpha}\right)\right]$$

where $i=1,2,\ldots,8$.

Proof. As it is known from lemma 2. 1, solution of the model (1.1) is solution of the integral (2.1) as well. Moreover, the fixed points of the operator $S: C^{\alpha}[0,T_0]\to C^{\alpha}[0,T_0]$ defined by

$$S\omega_i(t)=\frac{b_i}{\Gamma(\alpha)}x^{\alpha}+\frac{1}{\Gamma(\alpha)}\int_0^x \frac{f_i(t,X(t))}{(x-t)^{1-\alpha}}dt$$

Then let $\alpha=\frac{1}{2}$ and

$$f_1=-\beta\frac{x_3}{N}x_1-\iota\beta\frac{x_6}{N}x_1-\acute{\beta}\frac{x_4}{N}x_1$$

And

$$b_1=0$$

so,

$$S\omega_1(t)=\frac{b_1}{\Gamma(\alpha)}x^{\alpha}+\frac{1}{\Gamma(\alpha)}\int_0^x \frac{f_1(t,X(t))}{(t-x_1)^{1-\alpha}}dx_1$$

$$S\omega_1(t)=\frac{0}{\Gamma\left(\frac{1}{2}\right)}t^{\frac{1}{2}}+\frac{1}{\Gamma\left(\frac{1}{2}\right)}\int_0^x \frac{-\beta\frac{x_3}{N}x_1-\iota\beta\frac{x_6}{N}x_1-\acute{\beta}\frac{x_4}{N}x_1}{(t-x_1)^{1-\frac{1}{2}}}dx$$

$$= 0 + \frac{1}{\Gamma(\frac{1}{2})}\int_0^t \frac{(-\beta\frac{x_3}{N}x_1 - \iota\beta\frac{x_6}{N}x_1 - \acute{\beta}\frac{x_4}{N}x_1)}{(t-x_1)^{\frac{1}{2}}}dx_1$$

$$= \frac{(-\beta\frac{x_3}{N} - \iota\beta\frac{x_6}{N} - \acute{\beta}\frac{x_4}{N})}{\sqrt{\pi}}\int_0^t x_1(t-x_1)^{-\frac{1}{2}}dx_1$$

in the equation let $\beta\frac{x_3}{N} + \iota\beta\frac{x_6}{N} + \acute{\beta}\frac{x_4}{N} = M$ such that

$$= \frac{M}{\sqrt{\pi}}\int_0^t -x_1(t-x_1)^{-\frac{1}{2}}dx_1$$

$$= \frac{M}{\sqrt{\pi}}\int_0^t -x_1(t-x_1)^{-\frac{1}{2}}dx_1$$

$$= -\frac{4M}{3\sqrt{\pi}}t^{\frac{3}{2}}$$

Therefore, let $f_2 = \beta\frac{x_3}{N}x_1 + \iota\beta\frac{X_6}{N}x_1 + \acute{\beta}\frac{X_4}{N}x_1 - \kappa x_2$ and $b_2 = -\frac{2t^{\frac{1}{2}}}{\sqrt{\pi}}\kappa$ such that

$$S\omega_2(t) = \frac{b_i}{\Gamma(\alpha)}t^\alpha + \frac{1}{\Gamma(\alpha)}\int_0^x \frac{f_i(t, X(t))}{(t-x_2)^{1-\alpha}}dx_2$$

$$S\omega_2(t) = \frac{-\frac{\kappa}{\sqrt{\pi}}2t^{\frac{1}{2}}}{\Gamma(\frac{1}{2})}t^{\frac{1}{2}} + \frac{1}{\Gamma(\frac{1}{2})}\int_0^t \frac{(\beta\frac{x_3}{N}x_1 + \iota\beta\frac{x_6}{N}x_1 + \acute{\beta}\frac{x_4}{N}x_1 - \kappa x_2)}{(t-x_2)^{\frac{1}{2}}}dx_2$$

$$= \frac{-\frac{\kappa}{\sqrt{\pi}}2t^{\frac{1}{2}}}{\sqrt{\pi}}t^{\frac{1}{2}} + \frac{(\beta\frac{x_3}{N}x_1 + \iota\beta\frac{x_6}{N}x_1 + \acute{\beta}\frac{x_4}{N}x_1)}{\sqrt{\pi}}\int_0^t (t-x_2)^{-\frac{1}{2}}dx_2$$
$$+ \frac{\kappa}{\sqrt{\pi}}\int_0^t -x_2(t-x_2)^{-\frac{1}{2}}dx_2$$

then in the equation let $J = (\beta\frac{x_3}{N}x_1 + \iota\beta\frac{X_6}{N}x_1 + \acute{\beta}\frac{X_4}{N}x_1)$ such that

$$= -\frac{2t\kappa}{\pi} + \frac{J}{\sqrt{\pi}} \int_0^t (t - x_2)^{-\frac{1}{2}} dx_2 + \frac{\kappa}{\sqrt{\pi}} \int_0^t -x_2 (t - x_2)^{-\frac{1}{2}} dx_2$$

$$= -\frac{2t\kappa}{\pi} + \frac{J}{\sqrt{\pi}} 2t^{\frac{1}{2}} + \frac{\kappa}{\sqrt{\pi}} 2 \int_0^t (t - x_2)^{\frac{1}{2}} dx_2$$

$$= -\frac{2t\kappa}{\pi} + \frac{J}{\sqrt{\pi}} 2t^{\frac{1}{2}} - \frac{4}{3} \frac{\kappa}{\sqrt{\pi}} t^{\frac{4}{3}}.$$

So that, for similarly using same way for solving $S\omega_2, \dots S\omega_7$ and $S\omega_8$.

Therefore, are solutions of integral equation. For this goal, be a necessary to prove the operator S states at least one fixed point. For this, it will be shown that operator S satisfies the hypotheses of Schauder fixed point theorem. Let be a start with showing this following inclusion to be.

$$S(\mathcal{B}_r) \subset \mathcal{B}_r$$

$$\mathcal{B}_r = \{\omega_i \in C^\alpha[0, T] \colon \|\omega_i\|_\infty + \|D^\alpha \omega_i - b_i\|_\infty \le n, i = 1,2, \dots ,8\}$$

be a closed compact subset of the $C^\alpha[0, T]$. Accordingly, to a norm on $C^\alpha[0, T]$, upper bounded of $\|S\omega_i\|_\infty$ and $\|D^\alpha S\omega_i - b_i\|_\infty$ can be determined as follows:

$$|S\omega_i(t)| \le \left| \frac{b_i}{\Gamma(\alpha)} t^\alpha + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f_i(t, X(t))}{(t - x_i)^{1-\alpha}} dx_i \right|$$

$$= \frac{|b_i|}{\Gamma(\alpha)} t^\alpha + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{|f_i(t, X(t))|}{(t - x_i)^{1-\alpha}} dx_i$$

$$= \frac{|b_i|}{\Gamma(\alpha)} t^\alpha + \frac{m}{\Gamma(\alpha)} \int_0^t \frac{1}{(t\tau)^\alpha (t - t\tau)^{1-\alpha}} t d\tau$$

$$\le \frac{|b_i|}{\Gamma(\alpha)} t^\alpha + \frac{m}{\Gamma(\alpha)} \int_0^1 \frac{t}{\tau^\alpha (1-\tau)^\alpha} d\tau \le \frac{|b_i|}{\Gamma(\alpha)} t^\alpha + \Gamma(1-\alpha) mt \qquad (2.2)$$

And let $D^\alpha S\omega_i = b_i + \int_0^x f_i(t, X(t)) dt$ such that

$$|D^\alpha S\omega_i - b_i| = \left| b_i + \int_0^x f_i(t, X(t)) dt - b_i \right|$$

$$= \left| \int_0^x f_i\big(t, X(t)\big) dt \right| \le \int_0^x \frac{\left| t^\alpha f_i\big(t, X(t)\big) \right|}{t^\alpha} dtmx \int_0^\tau \tau^{1-\alpha} d\tau = \frac{mt^{1-\alpha}}{1-\alpha}$$

(2.3)

From (2.3) and (2.2) such that

$$|S\omega_i(t)| + |D^\alpha S\omega_i - b_i| = \frac{|b_i|}{\Gamma(\alpha)} t^\alpha + \Gamma(1-\alpha)mt + \frac{mt^{1-\alpha}}{1-\alpha}$$

that is gotten. Pleasing supremum over $[0, T]$ for a $T_0 > 0$ for the right hand-side of the above equation,

$$|S\omega_i(t)| + |D^\alpha S\omega_i - b_i| \leq C(b_i, \alpha, m)T_0^\alpha$$

Can be written, where $\alpha \in \Psi = \{\alpha, 0{,}1-\alpha\}$. α depends on values of b_i, α, m, n. To determine and T_0 and α let

$$C(b_i, \alpha, m)T_0^\alpha = n.$$

If $T_0^\alpha = \frac{n}{C(b_i,\alpha,m)} < 0$, then it is observed that $T_0 < 0$ for any $\alpha \in \Psi$. If $T_0^\alpha = \frac{n}{C(b_i,\alpha,m)} \geq 0$, it must be $T_0 \geq 0$ for any $\alpha \in \Psi$. Thus,

$$\sup[|S\omega_i(t)| + |D^\alpha S\omega_i - b_i|] \leq C(b_i, \alpha, m)T_0^\alpha = n,$$

where

$$T_0 = [\frac{n}{C(b_i, \alpha, m)}]^{1/\alpha}$$

and

$$\alpha = \begin{cases} 0 & if \quad \frac{n}{C(b_i, \alpha, m)} \geq 0 \\ \alpha & if \quad \frac{n}{C(b_i, \alpha, m)} < 0 \;\; and \;\; 0 < \alpha \leq 0.5 \\ 1-\alpha & if \quad \frac{n}{C(b_i, \alpha, m)} < 0 \;\; and \;\; 0.5 \leq \alpha < 1 \end{cases}$$

Consequently, for all cases we obtain

$$\|\omega_i\|_\infty + \|D^\alpha \omega_i - b_i\|_\infty \leq n,$$

which is the desired result.

We'll prove the equicontinuity of $S(\mathcal{B}_r) \subset C^\alpha[0, T]$. Since the composition of uniformly continuous function is so as well, the function $t^\alpha f_i(t, X(t))$ is uniformly continuous on $[0, T_0]$. Since for any $\omega_i \in \mathcal{B}_r$, both $\omega_i(t)$ and $D^\alpha \omega_i(t)$ and$x^\alpha f(t, u_1, u_2, \ldots, u_8)$ are uniformly continuous on I, respectively. Consequently, for given any $\epsilon > 0$, one can find a $\delta = \delta(\epsilon) > 0$ so that for all $t_1, t_2 \in [0, T_0]$ with $|t_1 - t_2| < \delta$ it is

$$|t^\alpha f_i(t_1, X(t_1)) - t^\alpha f_i(t_2, X(t_2))\ | < K\epsilon$$

Where $K = \max(\frac{1}{T_0\Gamma(1-\alpha)}, \frac{1-\alpha}{T^{1-\alpha}{}_0})$. It follows that

$$|S\omega_i(t_1) - S\omega_i(t_2)| + |D^\alpha S\omega_i(t_1) - D^\alpha S\omega_i(t_2)|$$

$$= \left|\frac{b_i}{\Gamma(\alpha)}x^{\boldsymbol{\alpha}} + \frac{1}{\Gamma(\alpha)}\int_0^x \frac{f_i\big(t_1, X(t_1)\big)}{(x-t_1)^{1-\alpha}}dt_1 - \frac{b_i}{\Gamma(\alpha)}x^{\boldsymbol{\alpha}} - \frac{1}{\Gamma(\alpha)}\int_0^x \frac{f_i(t_2, X(t_2))}{(x-t_2)^{1-\alpha}}dt_2\right|$$

$$+ \left|b_{\boldsymbol{i}} + \int_{\boldsymbol{0}}^{\boldsymbol{x}} f_i\big(t_1, X(t_1)\big)dt_1 - b_{\boldsymbol{i}} - \int_{\boldsymbol{0}}^{\boldsymbol{x}} f_i\big(t_2, X(t_2)\big)dt_2\right|$$

$$= \left|\frac{1}{\Gamma(\alpha)}\int_0^x \frac{f_i\big(t_1, X(t_1)\big)}{(x-t_1)^{1-\alpha}}dt_1 - \frac{1}{\Gamma(\alpha)}\int_0^x \frac{f_i\big(t_2, X(t_2)\big)}{(x-t_1)^{1-\alpha}}dt_2\right|$$

$$+ \left|\int_{\boldsymbol{0}}^{\boldsymbol{x}} f_i\big(t_1, X(t_1)\big)dt_1 - \int_{\boldsymbol{0}}^{\boldsymbol{x}} f_i\big(t_2, X(t_3)\big)dt_2\right|$$

$$= \left|\frac{1}{\Gamma(\alpha)}\int_0^x \frac{t^\alpha f_i\big(t_1, X(t_1)\big)}{(x-t_1)^{1-\alpha}}dt_1 - \frac{1}{\Gamma(\alpha)}\int_0^t \frac{t^\alpha f_i\big(t_2, X(t_2)\big)}{(x-t_1)^{1-\alpha}}dt_2\right|$$

$$+ \left|\int_{\boldsymbol{0}}^{\boldsymbol{x}} f_i\big(t_1, X(t_1)\big)dt_1 - \int_{\boldsymbol{0}}^{\boldsymbol{x}} f_i\big(t_1, X(t_2)\big)dt_2\right|$$

$$\leq \int_0^1 \frac{|h(\eta t_1) - h(\eta t_2)|}{\Gamma(\alpha)\eta^{1-\alpha}(1-\eta)^{1-\alpha}}td\eta + \int_0^1 \frac{|h(\eta t_1) - h(\eta t_2)|}{\eta^{1-\alpha}}t^{1-\alpha}d\eta$$

$$< T_0\Gamma(1-\alpha)K\epsilon + \frac{T_0^{1-\alpha}}{1-\alpha}K\epsilon = \epsilon,$$

where $h(t_j) = t^\alpha f_i\left(t_j, X\big(t_j\big)\right))$. Consequently, that $S(\mathcal{B}_r)$ is an equicontinuous set of $C^\alpha[0, T_0]$.

Finally, the continuity of S on $\mathcal{B}_r$ will be proven. Assume that $\{\omega_{ik}\}_{k=1}^\infty \subset \mathcal{B}_r$ is a sequence with $\omega_{ik} \xrightarrow{C^\alpha[0,T_0]} \omega_i$ as $k \to \infty$. Then one can easily conclude that ω_k and $D^\alpha\omega_{ik}(t)$ converges uniformly ω_i to $D^\alpha\omega_i(t)$, respectively. With these and the uniformly continuity of $t^\alpha f_i\big(t, u_1,\ u_{2\ ,} \dots, u_8\big)$ on I, it leads to

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$$\|S\omega_{ik} - S\omega_i\|_\infty$$

$$= \sup_{t\in[0,T_0]} \left| \frac{b_i}{\Gamma(\alpha)} x^{\boldsymbol{\alpha}} + \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f_i(t_k, X(t_k))}{(x-t)^{1-\alpha}} dt - \frac{b_i}{\Gamma(\alpha)} x^{\boldsymbol{\alpha}} - \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f_i(t, X(t))}{(x-t)} dt \right|$$

$$+ \sup_{t\in[0,T_0]} \left| b_{\boldsymbol{i}} + \int_{\boldsymbol{0}}^{\boldsymbol{t}} f_i(t_k, X(t_k)) dt - b_{\boldsymbol{i}} - \int_{\boldsymbol{0}}^{\boldsymbol{t}} f_i(t, X(t)) dt \right|$$

$$= \sup_{t\in[0,T_0]} \left| \frac{1}{\Gamma(\alpha)} \int_0^t \frac{[f_i(x_k, X(t_k)) - f_i(t, X(t))]}{(x-t)^{1-\alpha}} dt \right|$$

$$+ \sup_{t\in[0,T_0]} \left| \int_{\boldsymbol{0}}^{\boldsymbol{t}} [f_i(t_k, X(t_k)) - f_i(t, X(t))] dt \right|$$

$$\le \sup_{t\in[0,T_0]} \int_0^t \frac{(\eta t)^\alpha |f_i(\eta t, X(\eta t)) - f_i(\eta t, X(\eta t))|}{\Gamma(\alpha)(t-\eta)^\alpha \eta^\alpha} t d\eta$$

$$+ \sup_{t\in[0,T_0]} \int_0^t \frac{(\eta t)^\alpha |f_i(\eta t, X(\eta t)) - f_i(\eta t, X(\eta t))|}{\eta^\alpha} t^{1-\alpha} d\eta$$

$\to 0$ as $k \to \infty$.

In decision, subsequently hypotheses of the theorem 2. 1 are satisfied, it obtains that operator S admits at least one critical point in $C^\alpha[0, T_0]$, which is a solution of the model (1.1) as well.

2.2 Uniqueness

We show that the solution of the model is uniqueness and we show that by the following theorem before this theorem using the lemma.

Lemma 2.2.1. [7] Let $\alpha \in (0,1)$ and $\omega_i \in C^\alpha[0, T]$. Then there is a function $\mu: [0, T] \to [0, T]$ with $0 < \mu(t) < t$ so that

$$\omega_i(t) = D^\alpha \omega_i(0) \frac{t^\alpha}{\Gamma(\alpha)} + \Gamma(1-\alpha) (\mu(t))^\alpha D^\alpha \omega_i(\mu(t)),$$

is fulfilled.

This lemma can be showed by the following method which is used in ([7]) and we ignore it here. So, by used lemma can be solving the nagamo type uniqueness

Theorem 2.2.2 (Nagumo type uniqueness) Let $0 < \alpha < 1, 0 < T < \infty$ and the condition (C1) be fulfilled. Additionally, we assume that there exist a positive real number $L \leq \frac{1-\alpha}{T(1+\Gamma(2-\alpha)}$ such that the inequality

$$x^{\alpha}|f(t_1, X(t_1) - f(t_2, X(t_2)| \leq L \ \ sum \ \left|t_{i,1} - t_{i,2}\right|$$

Is satisfied all $x \in [0, T]$ and all $t_{i1}, t_{i2} \in \mathbb{R}$ with $i = 1,2, \dots, 8$. Then, (1.1) has at most one solution in the space $C^{\alpha}[0, T]$.

Proof. By the previous theorem we have showed the existence of the solution for model (1.1) For uniqueness: at the beginning we assume that we have two different solutions in the model (1.1) such as Ω_i and Ω_2 in the space $C^{\alpha}[0, T]$. Define of the function $\varphi(x)$ such that

$$\varphi(x) = \{|\omega_{1i} - \omega_{2i}| + |D^{\alpha}\omega_{1i} - D^{\alpha}\omega_{2i}|\}$$

let $\omega_{1i}, \omega_{2i} \in C^{\alpha}[0, T]$, the continuity $\varphi(x)$ on the $x \in (0, T]$ at $x = 0$ such that

$$0 \leq \lim_{x \to 0} \varphi(x) = \lim_{x \to 0} \{|\omega_{1i} - \omega_{2i}| + |D^{\alpha}\omega_{1i} - D^{\alpha}\omega_{2i}|\}$$

By the theorem in the

$$\omega_{ji}(t) = \frac{b_i}{\Gamma(\alpha)} x^{\alpha} + \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f_i(t, X(t))}{(x - t)^{1-\alpha}} dt \, , \qquad j = 1,2$$

and

$$D^{\alpha}\omega_{ji}(t) = b_i + \int_0^x f_i\big(t, X(t)\big) dt \, , \qquad j = 1,2$$

this implies that

$$\lim_{x \to 0} \varphi(x) = \lim_{x \to 0} \{|\omega_{1i} - \omega_{2i}| + |D^{\alpha}\omega_{1i} - D^{\alpha}\omega_{2i}|\}$$

$$= \lim_{x \to 0} \left\{ \left| \frac{b_i}{\Gamma(\alpha)} x^{\alpha} + \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f_i(t_1, X(t_1))}{(x - t)^{1-\alpha}} dt - \frac{b_i}{\Gamma(\alpha)} x^{\alpha} - \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f_i(t_2, X(t_2))}{(x - t)^{1-\alpha}} dt \right| + \lim_{x \to 0} \left| b_i + \int_0^x f_i\big(t_1, X(t_1)\big) dt - b_i - \int_0^x f_i\big(t_2, X(t_2)\big) dt \right| \right\}$$

$$= \lim_{x\to 0}\left\{\left|\frac{1}{\Gamma(\alpha)}\int_0^x \frac{f_i\big(t_1,X(t_1)\big)}{(x-t)^{1-\alpha}}dt - \frac{1}{\Gamma(\alpha)}\int_0^x \frac{f_i\big(t_2,X(t_2)\big)}{(x-t)^{1-\alpha}}dt\right| + \lim_{x\to 0}\left|\int_0^x f_i\big(t_1,X(t_1)\big)dt - \int_0^x f_i\big(t_2,X(t_2)\big)dt\right|\right\}$$

$$= \lim_{x\to 0}\left\{\frac{1}{\Gamma(\alpha)}\left|\int_0^x \frac{f_i\big(t_1,X(t_1)\big) - f_i\big(t_2,X(t_2)\big)}{(x-t)^{1-\alpha}}dt\right| + \lim_{x\to 0}\left|\int_0^x f_i\big(t_1,X(t_1)\big) - f_i\big(t_2,X(t_2)\big)dt\right|\right\}$$

$$= \lim_{x\to 0}\frac{1}{\Gamma(\alpha)}\int_0^x \frac{\left|t^\alpha f_i\big(t_1,X(t_1)\big) - f_i\big(t_2,X(t_2)\big)\right|}{t^\alpha(x-t)^{1-\alpha}}dt + \lim_{x\to 0}\int_0^x \frac{\left|t^\alpha\left(f_i\big(t_1,X(t_1)\big) - f_i\big(t_2,X(t_2)\big)\right)\right|dt}{t^\alpha}$$

Let $H_i\left(t_j,\omega_{ji}(t_j)\right) = t^\alpha f_i(t_j,X(t_j))$ such that

$$= \lim_{x\to 0}\frac{1}{\Gamma(\alpha)}\int_0^x \frac{\left|H_i\big(t_1,X(t_1)\big) - H_i\big(t_2,X(t_2)\big)\right|}{t^\alpha(x-t)^{1-\alpha}}dt + \lim_{x\to 0}\int_0^x \frac{\left|H_i\big(t_1,X(t_1)\big) - H_i\big(t_2,X(t_2)\big)\right|dt}{t^\alpha}$$

Assume that $t = \eta x$ and $dt = x d\eta$ such that

$$= \lim_{x\to 0}\frac{1}{\Gamma(\alpha)}\int_0^1 \frac{\left|K_i\big(\eta x,\omega_{1i}(\eta x)\big) - K_i\big(\eta x,\omega_{2i}(\eta x)\big)\right|}{(\eta x)^\alpha(x-\eta x)^{1-\alpha}}x d\eta + \lim_{x\to 0}\int_0^x \frac{|K_i(\eta x,\omega_{1i}(\eta x)) - K_i(\eta x,\omega_{2i}(\eta x))|}{(\eta x)^\alpha}x d\eta$$

$$\leq \lim_{x\to 0}\frac{1}{\Gamma(\alpha)}\int_0^x \frac{|K_i(\eta x,\omega_{1i}(\eta x)) - K_i(\eta x,\omega_{2i}(\eta x))|}{\eta^\alpha(1-\eta)^{1-\alpha}}d\eta + \lim_{x\to 0}\int_0^x \frac{|K_i(\eta x,\omega_{1i}(\eta x)) - K_i(\eta x,\omega_{2i}(\eta x))|}{\eta^\alpha}x^{1-\alpha}d\eta = 0$$

then use the condition (C1), respectively. Consequently $\lim_{x\to 0}\varphi(x)=0=\varphi(0)$. The fact that $\varphi(x)\geq 0$ on $[0,T]$ allows choose the point $x_0\in(0,T]$ such that

$$0<\varphi(x_0)=|\omega_{1i}(x_0)-\omega_{2i}(x_0)|+|D^{\alpha}\omega_{1i}(x_0)-D^{\alpha}\omega_{2i}(x_0)|$$

By mean value theorem in this lemma 3.3.2.

$$\begin{aligned}|\omega_{1i}(x_0)-\omega_{2i}(x_0)|\\ &=\left|\frac{b_i}{\Gamma(\alpha)}x_0^{\alpha}+\Gamma(1-\alpha)(x_0)^{\alpha}D^{\alpha}\omega_{1i}(x_0)-\frac{b_i}{\Gamma(\alpha)}x_0^{\alpha}\right.\\ &\left.-\Gamma(1-\alpha)(x_0)^{\alpha}D^{\alpha}\omega_{2i}(x_0)\right|\\ &=|\Gamma(1-\alpha)(x_0)^{\alpha}D^{\alpha}\omega_{1i}(x_0)-\Gamma(1-\alpha)(x_0)^{\alpha}D^{\alpha}\omega_{2i}(x_0)|\\ &=\Gamma(1-\alpha)(x_0)^{\alpha}|D^{\alpha}\omega_{1i}(x_0)-D^{\alpha}\omega_{2i}(x_0)|\end{aligned}$$

So that,

$$\begin{aligned}&=\Gamma(1-\alpha)x_0\left|(x_{0,1})^{\alpha}D^{\alpha}(\omega_{1i}-\omega_{2i})(x_{0,1})\right|+|D^{\alpha}\omega_{1i}(x_0)-D^{\alpha}\omega_{2i}(x_0)|\\ &=\Gamma(1-\alpha)x_0(x_{0,1})^{\alpha}\left|\left(f_i\left(x_{0,1},\omega_{1i}(x_{0,1})\right)-f_i\left(x_{0,1},\omega_{2i}(x_{0,1})\right)\right)\right|\end{aligned} \tag{2.4}$$

where $x_{0,1}\in(0,x_0)$.

Secondly, for the estimation of $|D^{\alpha}\omega_{1i}(x_0)-D^{\alpha}\omega_{2i}(x_0)|$ such that

$$\begin{aligned}|D^{\alpha}\omega_{1i}(x_0)-D^{\alpha}\omega_{2i}(x_0)|&=\left|b_i+\int_0^x f_i(t,X(t))dt-b_i-\int_0^x f_i(t,X(t))dt\right|\\ &=\left|\int_0^x f_i(t,X(t))dt-\int_0^x f_i(t,X(t))dt\right|\\ &=\int_0^x\frac{t^{\alpha}\left|\left(f_i(t,X(t))-f_i(t_2,X(t))\right)\right|}{t^{\alpha}}dt\\ &=\frac{x_0^{1-\alpha}}{1-\alpha}x_{0,2}^{\alpha}\left|\left(f_i\left(x_{0,2},\omega_{1i}(x_{0,2})\right)-f_i\left(x_{0,2},\omega_{2i}(x_{0,2})\right)\right)\right|\end{aligned} \tag{2.5}$$

where $x_{0,2}\in(0,x_0)$.

We specify x_1 as one of the point $x_{0,1}$and $x_{0,2}$ so that

$$\left|\left(H_i(x_1,\omega_{1i}(x_1)) - H_i(x_1,\omega_{2i}(x_1))\right)\right| \coloneqq \max\left(\left|\left(H_i(x_{0.1},\omega_{1i}(x_{0.1})) - H_i(x_{0.1},\omega_{2i}(x_{0.1}))\right)\right|, \left|\left(H_i(x_{0.2},\omega_{1i}(x_{0.2})) - H_i(x_{0.2},\omega_{2i}(x_{0.2}))\right)\right|\right)$$

This implies that from (2.5) and (2.4) we get

$$0 < \Phi(x_0) \leq \left(\Gamma(1-\alpha)x_0 + \frac{x_0^{1-\alpha}}{1-\alpha}\right)\left|\left(H_i(x_1,\omega_{1i}(x_1)) - H_i(x_1,\omega_{2i}(x_1))\right)\right|$$

$$\leq T\left(\frac{1+\Gamma(2-\alpha)}{1-\alpha}\right)x_1^{\alpha}\left|\left(f_i(x_1,\omega_{1i}(x_1)) - f_i(x_1,\omega_{2i}(x_1))\right)\right|$$

$$\leq T\mathrm{L}\left(\frac{1+\Gamma(2-\alpha)}{1-\alpha}\right)x_1^{\alpha}|\omega_{1i}-\omega_{2i}| + |D^{\alpha}\omega_{1i} - D^{\alpha}\omega_{2i}| = \Phi(x_1)$$

since $\mathrm{L} \leq \left(\frac{1-\alpha}{T(1+\Gamma(2-\alpha))}\right)$. Repeating the same procedure for the point x1, it enables us to find some points $x_2 \in (0,x_1)$ so that $0 < \Phi(x_0) \leq \Phi(x_1) \leq \Phi(x_2)$. Continuing in the same way, the sequence $\{x_n\}_{n=1}^{\infty} \subset [0,x_0)$ can be constructed so that $x_n \to 0$ and

$$0 < \Phi(x_0) \leq \Phi(x_1) \leq \Phi(x_2) \leq \cdots < \Phi(x_n) \leq \cdots \qquad (2.6)$$

However, the fact that $\Phi(x)$ is continuous at $x = 0$ and $x_n \to 0$ leads to $\Phi(xn) \to \Phi(0) = 0$, and this contradicts with (2.6). Consequently, IVP model (1.1) possesses a unique solution.

Conclusion

In this work, we present the dynamics of COVID-19 mathematical models which proposed by Ndaïrou et al in [1]. For better understand that this biological phenomenon that has appeared in China in recent years after that spread throughout the world has led to the death of millions of people. In this work we proved that the models have the existence and uniqueness solutions, which showing and proving some theorems. These theories help us to establish the patient's condition and its recovery in a hurry. This existence and uniqueness we help that to prove the model has the stability.

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