Academic Science Journal

On The Stability for Covid-19 Model

Jwan S. Ali and Hero W. Salih

Mathematics Department – Salahaddin University-Erbil

hero.salih@su.edu.krd

Received: 5 October 2022 **Accepted: 16 January 2023**

DOI: https://doi.org/10.24237/ASJ.02.01.702B

Abstract

In this paper, we investigate the stability of an impulsive mathematical model for a biological phenomenon that caused millions of people to die in these recent years, and it is the phenomenon of the COVID-19 epidemic, which first appeared in Wuhan, China. In this study we are working on the system that was proposed by Ndaïrou et al [1] to define the dynamics of COVID-19 model. For the stabilization study we use the direct and indirect Lyapunov method. Before we start a study, we must find all the critical points of the system. For more study, we perturbation the system by adding very small positive quantities, because in finding critical points we have three free variables in case of perturbation, can work on more points. It physically means that we can control a patient's condition and reduce the number of dead and injured recovering.

Keywords: stability, critical point, COVID-19 model, Lyapunov direct method,

دراسة الاستقرارية لمنظومة كوفيد-١٩

جوان شوان على – هيرۆ ويسى صالح

قسم الرياضيات- كلية العلوم - جامعة صلاح الدين- اربيل

الخلاصة

في هذا العمل ، قمنا بدراسة نموذج رياضي ا اقترحه Ndaïrou et al [1] لوصف ديناميكيات نموذج COVID-19. لفهم هذه الظواهر البيولوجية بشكل أفضل، نقوم بدراسة استقرار نقاط التوازن. للقيام بذلك ، يتم استخدام طريقة Lyapunov غير

المباشرة لنقاط التوازن. لا تسمح لنا أي من نقاط التوازن التي تم الحصول عليها بدراسة استقرار COVID-19، ثم نقترح bifurcation للنموذج وإجراء دراسة للنظام bifurcation الذي يساهم في فهم أفضل للعمليات الكيميائية الحيوية الأساسية التي تحكم هذا النشاط للغاية علاج مضاد للفيروسات. هذا يدل على أن هذا النموذج الرياضي واقعي بدرجة كافية لصياغة تأثير هذا العلاج.

كلمات مفتاحية : نقطة التوازن ، الاستقرار ، نموذج COVID-19 ، طريقة Lyapunov المباشرة ، طريقة Lyapunov غير المباشرة

Introduction

In this research, we study the model of coronavirus COVID-19. A fractional compartmental mathematical model is used for the spread of the COVID-19 illness. The transmissibility of super-spreaders has received special attention. Coronavirus illness 2019 (COVID-19), the outbreak due to severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), has taken on pandemic proportions in 2020 affecting more than 1.5 million individuals in almost all countries [2], to combat the COVID-19 pandemic, an integrated science and multidisciplinary approach is required [3]. Mathematical and epidemiological simulations, in particular, are essential for forecasting, anticipating, and managing current and future outbreaks. The sickness was originally discovered in Wuhan, China's capital, in December 2019 and has since spread around the world, resulting in the continuing 2020 pandemics outbreak. Because of thousands of confirmed infections and thousands of deaths around the world, the COVID-19 pandemic is considered the most serious global threat. Report 503, 274 confirmed cumulative cases with 22,342 deaths by March 26, 2020. According to a World Health Organization report dated April 8, 2020, the numbers had risen to 1,353,361 verified cumulative cases with 79,235 deaths at the time of this revision [1, 4]. As a result, fractional derivatives, which have been widely employed to generate models of infectious diseases since they account for the memory effect, which is now larger due to the aforementioned mean of the five prior days of daily reported cases, appear to be acceptable. Estimates of COVID-19 infected persons, generated a priori using mathematical models, have aided in predicting the number of required beds for both hospitalized individuals and, in particular, intensive care units. In this research we study the existence and uniqueness of solution for fractional differential equations with Caputo fractional derivative of order α. Moreover, it is an incessant function.

The Proposed COVID-19 Fractional Mode

The active area of this study is nonlinear differential equations and, in some cases, acceptable to include the history of the processes. The model was proposing in [1] in fractional differential equation models, but in this research, we study non-fractional case then the model becomes as the form:

$$\frac{\partial X_1}{\partial t} = -\beta \frac{x_3}{N} x_1 - \iota\beta \frac{x_6}{N} x_1 - \acute{\beta} \frac{x_4}{N} x_1.$$

$$\frac{\partial X_2}{\partial t} = \beta \frac{x_3}{N} x_1 + \iota\beta \frac{x_6}{N} x_1 + \acute{\beta} \frac{x_4}{N} x_1 - \kappa x_2.$$

$$\frac{\partial X_3}{\partial t} = \kappa\rho_1 x_2 - (\gamma_a + \gamma_i + \delta_i) x_3.$$

$$\frac{\partial X_4}{\partial t} = \kappa\rho_2 x_2 - \left(\gamma_a + \gamma_i + \delta_p\right) x_4. \tag{1.1}$$

$$\frac{\partial X_5}{\partial t} = \kappa(1 - \rho_1 - \rho_2) x_2.$$

$$\frac{\partial X_6}{\partial t} = \gamma_a(x_3 + x_4) - (\gamma_r + \delta_h) x_6.$$

$$\frac{\partial X_7}{\partial t} = \gamma_i(x_3 + x_4) + \gamma_r x_6.$$

$$\frac{\partial X_8}{\partial t} = \delta_i x_3 + \delta_p x_4 + \delta_h x_6.$$

Where x_1 is susceptible individuals, exposed individuals x_2, symptomatic and infectious individuals (x_3), x_4 is super-spreaders individuals, x_5 is infectious but asymptomatic individuals, x_6 is hospitalized individuals, recovery individuals (x_7), and dead individuals (x_8) or fatality class. And $x_i(0) > 0$ for $i = 1,2, \dots, 8$, and where $N = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$.

So implies that the following parameters:

β	the human-to-human communication
$\acute{\beta}$	quantifies a high communication quantity due to super-spreaders
l	quantifies the comparative transmissibility of hospitalized patients
κ	rate at which a different leaf the visible class by becoming communicable
ρ_1	quantity of evolution from exposed class x_2 to symptomatic communicable class x_3
ρ_2	is a relation low rate at which exposed persons become super-speared
$1-\rho_1-\rho_2$	is the evolution from visible to symptomless class
γ_a	is the average rate at which symptomless and super-speared persons become hospitalized
γ_i	the repossession rate without being hospitalized
γ_r	repossession rate of hospitalized patients
δ_i	illness caused by death rates due to infected individuals
δ_p	illness caused by death rates due to super-spreads individuals
δ_h	illness caused by death rates due to hospitalized individuals.

If α is rational number then let $\alpha = 1$ we have to find the stability.

Stationary states

To investigate the stability of fractional differential system (1.1), we estimate all critical points of (1.1) and use the Lyapunov indirect method. To estimate the critical points let $(x_1, x_2,, x_3, x_4, x_5, x_6, x_7, x_8)$ be a stationary state of system (1.1). Then it satisfy $\dot{x} = 0$ for

$i = 1, 2, \ldots, 8$ that is say

$$0 = -\beta \frac{x_3}{N} x_1 - \iota\beta \frac{x_6}{N} x_1 - \acute{\beta} \frac{x_4}{N} x_1. \quad (1.2)$$

$$0 = \beta \frac{x_3}{N} x_1 + \iota\beta \frac{x_6}{N} x_1 + \acute{\beta} \frac{x}{N} x_1 - \kappa x_2. \quad (1.3)$$

$$0 = \kappa\rho_1 x_2 - (\gamma_a + \gamma_i + \delta_i) x_3. \quad (1.4)$$

$$0 = \kappa\rho_2 x_2 - (\gamma_a + \gamma_i + \delta_p) x_4. \quad (1.5)$$

$$0 = \kappa(1 - \rho_1 - \rho_2) x_2. \quad (1.6)$$

$$0 = \gamma_a(x_3 + x_4) - (\gamma_r + \delta_h) x_6. \quad (1.7)$$

$$0 = \gamma_i(x_3 + x_4) + \gamma_r x_6. \quad (1.8)$$

$$0 = \delta_i x_3 + \delta_p x_4 + \delta_h x_6. \quad (1.9)$$

To calculate the number of critical point of models. From (1.6)we have $x_2 = 0$ put in (1.4)and (1.5) we get $x_3 = x_4 = 0$. from (1.8) we get $x_6 = 0$and all of x_1, x_5, x_7, x_8 are free variables then $E_0 = (a, 0, 0, 0, b, 0, c, d)$ is the only critical point of the model with a, b, c, d are free variable and positive.

Local stability

To start this investigation, we need recall the following

Routh-Hurwitz Stability [5] a matrix $A \in R(n \times n)$ is called Hurwitz or asymptotically stable if and only if

$$Re(\lambda_i) < 0, \qquad \forall i = 1, 2, \ldots, n$$

Where λ_i 's are the eigenvalues of the matrix A.

(Indirect Method Theorem) Let $x = 0$ be an critical point for $x = f(x)$ where f is a continuously differentiable function from D, a neighborhood of the origin in to R_n. And

$$A = \left[\frac{\partial f}{\partial x}\right]|_{x=0}$$

then

The origin is asymptotically stable if $Re(\lambda_i) < 0$ for all eigenvalues of A.

The origin is unstable if $Re(\lambda_i) < 0$ for one or more of the eigenvalues of A.

In order to investigate the stability of system (1.2) -(1.9), the corresponding linearized system at the equilibrium point $E_0 = (a, 0, 0, 0, b, 0, c, d)$ is taken into account.

$$\dot{x_1} = -\beta \frac{a}{N} x_3 - \iota\beta \frac{a}{N} x_6 - \acute{\beta} \frac{a}{N} x_4.$$

$$\dot{x_2} = \beta \frac{a}{N} x_3 + \iota\beta \frac{a}{N} x_6 + \acute{\beta} \frac{a}{N} x_4 - \kappa x_2.$$

$$\dot{x_3} = \kappa\rho_1 x_2 - (\gamma_a + \gamma_i + \delta_i) x_3.$$

$$\dot{x_4} = \kappa\rho_2 x_2 - (\gamma_a + \gamma_i + \delta_p) x_4. \tag{1.10}$$

$$\dot{x_5} = \kappa(1 - \rho_1 - \rho_2) x_2.$$

$$\dot{x_6} = \gamma_a (x_3 + x_4) - (\gamma_r + \delta_h) x_6.$$

$$\dot{x_7} = \gamma_i (x_3 + x_4) + \gamma_r x_6.$$

$$\dot{x_8} = \delta_i x_3 + \delta_p x_4 + \delta_h x_6.$$

Which can be written as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\beta \frac{a}{N} & -\acute{\beta} \frac{a}{N} & 0 & -\iota\beta \frac{a}{N} & 0 & 0 \\ 0 & -\kappa & \beta \frac{a}{N} & \acute{\beta} \frac{a}{N} & 0 & \iota\beta \frac{a}{N} & 0 & 0 \\ 0 & \kappa\rho_1 & K_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa\rho_2 & 0 & K_2 & 0 & 0 & 0 & 0 \\ 0 & K_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_a & \gamma_a & 0 & K_4 & 0 & 0 \\ 0 & 0 & \gamma_i & \gamma_i & 0 & \gamma_r & 0 & 0 \\ 0 & 0 & \delta_i & \delta_p & 0 & \delta_h & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{pmatrix}$$

Where

$$K_1 = -(\gamma_a + \gamma_i + \delta_i),$$

$$K_2 = -(\gamma_a + \gamma_i + \delta_p),$$

$$K_3 = \kappa(1 - \rho_1 - \rho_2)$$

$$K_4 = -(\gamma_r + \delta_h)$$

The Jacobian matrix is the linearization system of system (1.10) at the critical point E_0. We proceed by using the Routh–Hurwitz criterion to determine conditions under which the Jacobian matrix possesses non-negative eigenvalues. The characteristic polynomial of this Jacobian matrix of may be represented as

$$P(\lambda) = a_8\lambda^8 + a_7\lambda^7 + \cdots + a_1\lambda + a_0$$

where λ denotes the eigenvalues. Then the Characteristic polynomial, of the (1.10) is

$$a(\lambda) = det(A - \lambda I_8)$$

$$= \lambda^4(-\kappa - \lambda)(-(\gamma_a + \gamma_i + \delta_p) - \lambda)(-(\gamma_a + \gamma_i + \delta_i) - \lambda)(-(\gamma_r + \delta_h) - \lambda) \quad (1.11)$$

Thus the eigenvalues are $\lambda = 0$ multiplicity 4 times; $\lambda = -(\gamma_a + \gamma_i + \delta_p)$;

$\lambda = -(\gamma_a + \gamma_i + \delta_i)$; $\lambda = -\kappa$; $\lambda = -(\gamma_r + \delta_h)$ Since these parameters are positive, so all the eigenvalues are negative or equal to zero. Then the zero solution is stable if and only if the eigenvalues of Jacobian matrix A have a real part $Re(\lambda) \leq 0$ and those with part $Re(\lambda) = 0$ are simple. then the system (1.1) are stabile in the origin. local stability of the system by using Lyapunov method for the fixed point E_0 we get that

$$V(x_1, x_2, \dots, x_8) = \sum_{i=0}^{8} \lambda_i (x_i)^2$$

If $\lambda_i = 1$ so implies that

$$V(x_1, x_2, \dots, x_8) = \sum_{i=0}^{8} {x_i}^2 = x_1^2 + x_2^2 + x_3^2 + \cdots + x_8^2$$

$$\dot{V}(x_1, x_2, \dots, x_8) = 2x_1\dot{x}_1 + 2x_2\dot{x}_2 + \dots + 2x_8\dot{x}_8$$

$$= 2x_1\left(-\beta\frac{x_3}{N}x_1 - \iota\beta\frac{x_6}{N}x_1 - \acute{\beta}\frac{x_4}{N}x_1\right) + 2x_2\left(\beta\frac{x_3}{N}x_1 + \iota\beta\frac{x_6}{N}x_1 + \acute{\beta}\frac{x_4}{N}x_1 - \kappa x_2\right)$$
$$+ 2x_3(\kappa\rho_1 x_2 - (\gamma_a + \gamma_i + \delta_i)x_3) + 2x_4(\kappa\rho_2 x_2 - (\gamma_a + \gamma_i + \delta_p)x_4$$
$$+ 2x_5(\kappa(1 - \rho_1 - \rho_2)x_2) + 2x_7(\gamma_a(x_3 + x_4) - (\gamma_r + \delta_h)x_6)$$
$$+ 2x_7(\gamma_i(x_3 + x_4) + \gamma_r x_6) + 2x_8(\delta_i x_3 + \delta_p x_4 + \delta_h x_6).$$

for this fixed point of the Lyapnouv is stable by theorem (Theorem 3.25. If there exists a function

$V(X)$ which is definite in V $*$ such that $V\ \dot{V}$ is continuous and negative semi-definite ($V\ \dot{V} \leq 0$); then the equilibrium point ($X = 0$) is stable.)

Note that, in all the critical points of this system, the component representing the susceptible individuals, exposed individuals infectious but asymptomatic individuals, dead individuals and fatality class are free variables. This does not make it imaginable to investigate the stable state around a positive value of this variable. In order to preparation this problem, we propose in the following section, based on the theory of bifurcation [[6],[7]]a investigation of stability by presenting a small perturbation of the variable representing the free variables.

Bifurcation of the mathematical model

Bifurcation theory refers to the investigation of qualitative changes to the state of a system as the parameters are varied. Else the bifurcation of differential equation is disturbed by changes in the qualitative behavior of its phase portrait as a set of parameters.

Perturbation theory observes parameter dependence of local solutions. To present elementary ideas simply, consider a four-parameter family of functions: for each x in a set $\mathfrak{R}$ and real parameter ϵ_i in a punctured neighborhood of $\epsilon = 0$, the values of the function $f(x, \epsilon)$ are in a metric space. The range is a metric space so that convergence of f as $\epsilon \to 0$ can be discussed. $f(x, \epsilon)$ is to be regarded as a solution of some set of equation containing ε as a parameter.

The equations are called a regularly perturbed problem if all solutions $f(x, \epsilon)$ converge uniformly on $\mathbb{R}$ as $\epsilon \to 0$.

If there is a solution which does not uniformly converge, the problem is called singularly perturbed. Let us bifurcate the model (1.1) by adding a small parameter $\epsilon_i, i = 1,2,3,4$ at the level of x_1, x_5, x_7, x_8, and $N = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + \varepsilon$ where $\sum \epsilon_i, \quad i = 1,2,3,4$ which results in the following model:

$$\dot{x}_1 = -\beta \frac{x_3}{N} x_1 - \iota\beta \frac{x_6}{N} x_1 - \acute{\beta} \frac{x_4}{N} x_1 - \epsilon_1 x_1.$$

$$\dot{x}_2 = \beta \frac{x_3}{N} x_1 + \iota\beta \frac{x_6}{N} x_1 + \acute{\beta} \frac{x_4}{N} x_1 - \kappa x_2.$$

$$\dot{x}_3 = \kappa\rho_1 x_2 - (\gamma_a + \gamma_i + \delta_i) x_3.$$

$$\dot{x}_4 = \kappa\rho_2 x_2 - (\gamma_a + \gamma_i + \delta_p) x_4. \quad (1.12)$$

$$\dot{x}_5 = \kappa(1 - \rho_1 - \rho_2) x_2 - \epsilon_2 x_5.$$

$$\dot{x}_6 = \gamma_a (x_3 + x_4) - (\gamma_r + \delta_h) x_6.$$

$$\dot{x}_7 = \gamma_i (x_3 + x_4) + \gamma_r x_6 - \epsilon_3 x_7.$$

$$\dot{x}_8 = \delta_i x_3 + \delta_p x_4 + \delta_h x_6 - \epsilon_4 x_8.$$

Stationary states

To investigate stability of system (1.11), we estimate all critical points of (2.11) and use Lyapunov indirect method. To estimate the critical points let $(x_1, x_2, , x_3, x_4, x_5, x_6, x_7, x_8)$ be a stationary state of system (1.11). To study stability, we must find the critical point of the model using the definition of critical point (In the general second-order linear homogeneous ordinary differential equation as an example

$$P(t)y'' + Q(t)y' + R(t)y = 0$$

The critical points of the ordinary differential equation are the values of t for which $P(t) = 0$ if P, Q and R are assumed to be polynomials and there is no common factor between them) this implies that this solution is critical point. Then it satisfies $\dot{x}_\iota = 0$ for $i = 1, 2, \ldots, 8$ then we obtain that $f_i(x_1, x_2,, x_3, x_4, x_5, x_6, x_7, x_8) = 0$ for $i = 1, 2, \dots, 8$ then the system (1.12) becomes as follows

$$0 = -\beta \frac{x_3}{N} x_1 - \iota\beta \frac{x_6}{N} x_1 - \acute{\beta} \frac{x_4}{N} x_1 - \epsilon_1 x_1.$$

$$0 = \beta \frac{x_3}{N} x_1 + \iota\beta \frac{x_6}{N} x_1 + \acute{\beta} \frac{x}{N} x_1 - \kappa x_2.$$

$$0 = \kappa\rho_1 x_2 - (\gamma_a + \gamma_i + \delta_i) x_3.$$

$$0 = \kappa\rho_2 x_2 - (\gamma_a + \gamma_i + \delta_p) x_4. \qquad (1.13)$$

$$0 = k(1 - \rho_1 - \rho_2) x_2 - \epsilon_2 x_5.$$

$$0 = \gamma_a (x_3 + x_4) - (\gamma_r + \delta_h) x_6.$$

$$0 = \gamma_i (x_3 + x_4) + \gamma_r x_6 - \epsilon_3 x_7.$$

$$0 = \delta_i x_3 + \delta_p x_4 + \delta_h x_6 - \epsilon_4 x_8.$$

Then this system (1.13) we have

$$x_3 = \frac{\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)} x_2$$

and

$$x_4 = \frac{\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_p)} x_2$$

Since $\gamma_i, \gamma_a, \delta_i$and δ_p positive everywhere then we obtain that $(\gamma_i + \gamma_a + \delta_i) \neq 0$ and

$(\gamma_i + \gamma_a + \delta_p) \neq 0$.

So, this system (1.13) we get

$$x_5 = \frac{\kappa(1 - \rho_1 - \rho_2)}{\varepsilon_2} x_2$$

And

$$x_6 = \frac{\gamma_a}{\gamma_r + \delta_h}\left(\frac{\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)} + \frac{\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_p)}\right)x_2$$

So implies that by this equation we have

$$x_7 = \left(\frac{\gamma_i}{\epsilon_3} + \frac{\gamma_r\gamma_a}{\epsilon_3(\gamma_r + \delta_h)}\right)\left(\frac{\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)} + \frac{\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_p)}\right)x_2$$

And

$$x_8 = \frac{1}{\epsilon_4}\left(\frac{\delta_i\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)} + \frac{\delta_p\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_p)} + \frac{\delta_h\gamma_a}{\gamma_r + \delta_h}\left(\frac{\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)} + \frac{\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_p)}\right)\right)x_2$$

From the system (2.11) we have $x_1(-Ax_2 + \epsilon_1) = 0$ where

$A = -\frac{\beta}{N}\frac{\kappa\rho_1}{(\gamma_i+\gamma_a+\delta_i)} - \frac{\iota\beta}{N}\frac{\gamma_a}{\gamma_r+\delta_h}\left(\frac{\kappa\rho_1}{(\gamma_i+\gamma_a+\delta_i)} + \frac{\kappa\rho_2}{(\gamma_i+\gamma_a+\delta_p)}\right) - \frac{\acute{\beta}}{N}\frac{\kappa\rho_2}{(\gamma_i+\gamma_a+\delta_p)} - \epsilon_1$ and $A \neq 0$ we get that or $x_1 = 0$ or $(-Ax_2 + \epsilon_1) = 0$

if $x_1 = 0$ then we $x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = x_8 = 0$ the $E_{b1} = (0,0,0,0,0,0,0,0)$ is the equilibrium point if $(-Ax_2 + \epsilon_1) = 0$ then $x_2 = \frac{\epsilon_1}{A}, A \neq 0$ then

$E_{b2} = (\frac{\kappa\delta_1}{A^2}, \frac{\epsilon_1}{A}, B_1, B_2, B_3, B_4, B_5, B_6)$ where

$$B_1 = \frac{\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)}\frac{\epsilon_1}{A}$$

$$B_2 = \frac{\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_i)}\frac{\epsilon_1}{A}$$

$$B_3 = \frac{\kappa(1 - \rho_1 - \rho_2)}{\varepsilon_2}\frac{\epsilon_1}{A}$$

$$B_4 = \frac{\gamma_a}{\gamma_r + \delta_h}\left(\frac{\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)} + \frac{\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_p)}\right)\frac{\epsilon_1}{A}$$

$$B_6 = \left(\frac{\gamma_i}{\epsilon_3} + \frac{\gamma_r\gamma_a}{\epsilon_3(\gamma_r + \delta_h)}\right)\left(\frac{\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)} + \frac{\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_p)}\right)\frac{\epsilon_1}{A}$$

$$B_6 = \frac{1}{\epsilon_4}\left(\frac{\delta_i\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)} + \frac{\delta_p\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_p)} + \frac{\delta_h\gamma_a}{\gamma_r + \delta_h}\left(\frac{\kappa\rho_1}{(\gamma_i + \gamma_a + \delta_i)} + \frac{\kappa\rho_2}{(\gamma_i + \gamma_a + \delta_p)}\right)\right)\frac{\epsilon_1}{A}$$

Local stability for bifurcate model

To investigate stability of system (1.12), the conforming linearized system at the critical point $E_{b1} = (0, 0, 0, 0, 0, 0, 0, 0)$ is taken into account

$$\dot{x}_1 = -\epsilon_1 x_1.$$

$$\dot{x}_2 = \beta\frac{x_3}{N}x_1 + \iota\beta\frac{x_6}{N}x_1 + \acute{\beta}\frac{x_4}{N}x_1 - \kappa x_2.$$

$$\dot{x}_3 = \kappa\rho_1 x_2 - (\gamma_a + \gamma_i + \delta_i)x_3.$$

$$\dot{x}_4 = \kappa\rho_2 x_2 - (\gamma_a + \gamma_i + \delta_p)x_4. \quad (1.14)$$

$$\dot{x}_5 = \kappa(1 - \rho_1 - \rho_2)x_2 - \epsilon_2 x_5.$$

$$\dot{x}_6 = \gamma_a(x_3 + x_4) - (\gamma_r + \delta_h)x_6.$$

$$\dot{x}_7 = \gamma_i(x_3 + x_4) + \gamma_r x_6 - \epsilon_3 x_7.$$

$$\dot{x}_8 = \delta_i x_3 + \delta_p x_4 + \delta_h x_6 - \epsilon_4 x_8.$$

Which can be written as:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\\ \dot{x}_7\\ \dot{x}_8\end{pmatrix}=\begin{pmatrix}-\epsilon_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -\kappa & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \kappa\rho_1 & -C_1 & 0 & 0 & 0 & 0 & 0\\ 0 & \kappa\rho_2 & 0 & C_2 & 0 & 0 & 0 & 0\\ 0 & C_3 & 0 & 0 & -\epsilon_2 & 0 & 0 & 0\\ 0 & 0 & \gamma_a & \gamma_a & 0 & C_4 & 0 & 0\\ 0 & 0 & \gamma_i & \gamma_i & 0 & \gamma_r & -\epsilon_3 & 0\\ 0 & 0 & \delta_i & \delta_p & 0 & \delta_h & 0 & -\epsilon_4\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\\ x_7\\ x_8\end{pmatrix} \quad (1.15)$$

where

$$C_1 = (\gamma_a + \gamma_i + \delta_i)$$

$$C_2 = -(\gamma_a + \gamma_i + \delta_p)$$

$$C_3 = \kappa(1 - \rho_1 - \rho_2)$$

$$C_4 = -(\gamma_r + \delta_h)$$

The Jacobian matrix is linear of system (1.11) at the critical point E_{b1}. We proceed by using the Routh–Hurwitz criterion to determine conditions under which the Jacobian matrix possesses non-negative eigenvalues.

The characteristic polynomial of this Jacobian matrix is represented as

$$P(\lambda) = a_8\lambda^8 + a_7\lambda^7 + \cdots + a_1\lambda + a_0$$

where λ denotes the eigenvalues. Then the Characteristic polynomial, of the (1.14) is

$$a(\lambda) = \det(A - \lambda I_8)$$

$$= (-\epsilon_1 - \lambda)(-\kappa - \lambda)(-(\gamma_a + \gamma_i + \delta_i) - \lambda)\big(-(\gamma_a + \gamma_i + \delta_p) - \lambda\big)(-(\gamma_r + \delta_h) - \lambda)(-\epsilon_2 - \lambda)(-\epsilon_3 - \lambda)(-\epsilon_4 - \lambda)$$

Thus the eigenvalues are $\lambda = -(\gamma_a + \gamma_i + \delta_i)$; $\lambda = -(\gamma_a + \gamma_i + \delta_p)$; $\lambda = -\kappa$;

$$\lambda = -(\gamma_r + \delta_h).,\lambda = -\epsilon_1, \lambda = -\epsilon_2, \lambda = -\epsilon_3 \; and \; \lambda = -\epsilon_4$$

Since these parameters are positive, so all the eigenvalues are negative or equal to zero. Then the zero solution is stable if and only if the eigenvalues of Jacobian Matrix A have a real part

$$Re(\lambda) \leq 0$$

local stability of system (1.14), the corresponding linearized system at the equilibrium point $E_{b2} = (\frac{\kappa\epsilon_1}{A^2}, \frac{\epsilon_1}{A}, B_1, B_2, B_3, B_4, B_5, B_6)$ is taken into account where $A_1 = \frac{\kappa\epsilon_1}{A^2}$and $A_1 = \frac{\epsilon_1}{A}$, $A \neq 0$

$$\dot{x}_1 = \left(-\beta\frac{B_1}{N} - \iota\beta\frac{B_4}{N} - \acute{\beta}\frac{B_2}{N} - \epsilon_1\right)x_1.$$

$$\dot{x}_2 = (\beta\frac{B_1}{N} + \iota\beta\frac{B_4}{N} + \acute{\beta}\frac{B_2}{N})x_1 - \kappa x_2.$$

$$\dot{x}_3 = \kappa\rho_1 x_2 - (\gamma_a + \gamma_i + \delta_i)x_3.$$

$$\dot{x}_4 = \kappa\rho_2 x_2 - (\gamma_a + \gamma_i + \delta_p)x_4. \quad (1.16)$$

$$\dot{x}_5 = \kappa(1 - \rho_1 - \rho_2)x_2 - \epsilon_2 x_5.$$

$$\dot{x}_6 = \gamma_a(x_3 + x_4) - (\gamma_r + \delta_h)x_6.$$

$$\dot{x}_7 = \gamma_i(x_3 + x_4) + \gamma_r x_6 - \epsilon_3 x_7.$$

$$\dot{x}_8 = \delta_i x_3 + \delta_p x_4 + \delta_h x_6 - \epsilon_4 x_8.$$

Which can be written as:

$$\begin{pmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\\\dot{x}_4\\\dot{x}_5\\\dot{x}_6\\\dot{x}_7\\\dot{x}_8\end{pmatrix} = \begin{pmatrix} D_0 & 0 & D_1 & D_2 & 0 & D_3 & 0 & 0 \\ -D_8 & -\kappa & -D_1 & -D_2 & 0 & -D_3 & 0 & 0 \\ 0 & \kappa\rho_1 & D_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa\rho_2 & 0 & D_5 & 0 & 0 & 0 & 0 \\ 0 & D_6 & 0 & 0 & -\epsilon_2 & 0 & 0 & 0 \\ 0 & 0 & \gamma_a & \gamma_a & 0 & D_7 & 0 & 0 \\ 0 & 0 & \gamma_i & \gamma_i & 0 & \gamma_r & -\epsilon_3 & 0 \\ 0 & 0 & \delta_i & \delta_p & 0 & \delta_h & 0 & -\epsilon_4 \end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\\x_7\\x_8\end{pmatrix} \quad (1.17)$$

$$D_0 = -\beta\frac{B_1}{N} - \iota\beta\frac{B_4}{N} - \acute{\beta}\frac{B_2}{N} - \epsilon_1$$

$$D_1 = -\beta\frac{B_1}{N}, \qquad D_2 = \acute{\beta}\frac{B_2}{N}, \qquad D_3 = \iota\beta\frac{B_4}{N}$$

$$D_4 = -(\gamma_a + \gamma_i + \delta_i)$$

$$D_5 = -(\gamma_a + \gamma_i + \delta_p)$$

$$D_6 = \kappa(1 - \rho_1 - \rho_2)$$

$$D_7 = -(\gamma_r + \delta_h)$$

$$D_8 = \beta\frac{B_1}{N} + \iota\beta\frac{B_4}{N} + \acute{\beta}\frac{B_2}{N}$$

The matrix of this linear system is the Jacobian matrix of system (2.16) at the equilibrium point Eb1. We proceed by using the Routh–Hurwitz criterion to determine conditions under which the Jacobian matrix possesses non-negative eigenvalues.

The characteristic polynomial of this Jacobian matrix may be represented as

$$P(\lambda) = a_8\lambda^8 + a_7\lambda^7 + \ldots + a_1\lambda + a_0.$$

where λ denotes the eigenvalues. Then the Characteristic polynomial, of the (2.16) is

$$a(\lambda) = \det(A - \lambda I_8)$$

$$= \left(\left(-\beta\frac{B_1}{N} - \iota\beta\frac{B_4}{N} - \acute{\beta}\frac{B_2}{N} - \epsilon_1\right) - \lambda\right)(-\kappa - \lambda)(-(\gamma_a + \gamma_i + \delta_i) - \lambda)\big(-(\gamma_a + \gamma_i + \delta_p) - \lambda\big)(-(\gamma_r + \delta_h) - \lambda)(-\epsilon_2 - \lambda)(-\epsilon_3 - \lambda)(-\epsilon_4 - \lambda)$$

Thus the eigenvalues are $\lambda = \left(-\beta\frac{B_1}{N} - \iota\beta\frac{B_4}{N} - \acute{\beta}\frac{B_2}{N} - \epsilon_1\right)$; $\lambda = -(\gamma_a + \gamma_i + \delta_i)$;

$\lambda = -(\gamma_a + \gamma_i + \delta_p)$; $\lambda = -\kappa$; $\lambda = -(\gamma_r + \delta_h).$, $\lambda = -\epsilon_1, \lambda = -\epsilon_2, \lambda = -\epsilon_3$ and

$$\lambda = -\epsilon_4$$

Since these parameters are positive, so all the eigenvalues are negative or equal to zero. Then the zero solution is stable if and only if the eigenvalues of Jacobian matrix A have a real part

$Re(\lambda) \leq 0$ and those with part.

Conclusion

Classical model considers SIR populations. Here we have taken into consideration there (x_4) the supper-spreaders, (x_6) hospitalized and (x_8) fatality class, so that its derivative gives the number of deaths. Our model is an ad hoc compartmental model of the COVID-19, taking into account its particularities, some of them still not well-known. We propose the investigate stability of an impulsive mathematical model for a biological phenomenon that caused millions of people to have died in these recent years, and it is the phenomenon of the COVID-19 epidemic, which first appeared in Wuhan, China. In this study we are working on the system that proposed by Ndaïrou et al [1] to define the dynamics of COVID-19 model. For the stabilization study we use the direct and indirect Lyapunov method. Before we start a study, we must find all the critical points of the system. For more study we perturbation the system by adding very small positive quantities, because in finding critical points we have three free variables in case of perturbation we can work on more points. It physically means that we can control a patient's condition and reduce the number of dead and injured recovering. In both the case original model (2.1) and perturbation model (2.12). In (2.1) we fixed all the variables x_i, $i = 1,2, \ldots ,8$ we the critical points and the model become stable if (the average rate at symptomless, super-spread persons become hospitalized, repossession rate without being hospitalized and caused by death rates duo to infected individuals or caused by death rates duo to super-spreads individuals) are increased and in this case bifurcation where the model add some (ϵ) because in the original model we have all the susceptible individuals, infections but asymptomatic individuals, dead individuals and fatality class are free after that all use some method to study the stability. In two neighborhoods for the critical points (the average rate at symptomless, super-spread persons become hospitalized, repossession rate without being hospitalized and caused by death rates duo to infected individuals or caused by death rates duo to super-spreads individuals) with $(\epsilon_i), i = 1,2,3,4$ are increased.

References

1. Ndaïrou, F., et al., Fractional model of COVID-19 applied to Galicia, Spain and Portugal. Chaos, Solitons & Fractals, 144: p. 110652. (2021).
2. Csse, J., Coronavirus COVID-19 global cases by the center for systems science and engineering (CSSE) at Johns Hopkins University (JHU). Johns Hopkins University (JHU): Baltimore, MD, USA, 2020.
3. Moradian, N., et al., The urgent need for integrated science to fight COVID-19 pandemic and beyond. Journal of translational medicine,18(1):1-7 (2020).
4. Ndaïrou, F., et al., Mathematical modeling of COVID-19 transmission dynamics with a case study of Wuhan. Chaos, Solitons & Fractals,135:109846 (2020).
5. Erawaty, N. and A. Amir. Stability Analysis for Routh-Hurwitz Conditions Using Partial Pivot. in Journal of Physics: Conference Series. (2019). IOP Publishing.
6. Wiggins, S. and E. Zeidler, Introduction to applied nonlinear dynamical systems and chaos. Texts in Applied Mathematics, vol. 5 Springer-Verlag (1990), New York, Berlin, Heidelberg, London, Paris, Tokyo, Hong Kong, 291 Illustrations, Subject Index, Bibliography, 670 pages. DM 98, 00, ISBN 0-387-97003-7 Springer-Verlag New York, Berlin, Heidelberg; ISBN 3-540-97003-7 Springer-Verlag Berlin, Heidelberg, New York. (1991), Wiley Online Library.
7. Nachaoui, A. and H. Salih, On the stability of a mathematical model for HIV (AIDS)-cancer dynamics. Mathematical Modeling and Computing, 8(4): 783-796. (2021).