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Efficient Solutions for Multi-criteria Sequencing Problem by Using Modified Algorithm

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Abstract

For multi criteria sequencing problem on one machine, we propose a modified branch and bound algorithm (MBAB) to find efficient (pareto optimal) solutions in this paper. The criteria are total completion time ($\sum C_j$), total lateness ($\sum L_j$), and maximum tardiness (T_{max}). A collection of n independent tasks(jobs) has to be sequenced on one machine , tasks(jobs)j (j=1,2,3,…..,n) requires processing time P_j and due data d_j . The MBAB algorithm depends on branch and bound technique. Applied examples are used to show applicability of MBAB algorithm. The MBAB algorithm is compared with complete enumeration method (CEM). Conclusions are formulated on the performance of the (MBAB) algorithm.

Keywords: Sequencing Multi-criteria, Efficient Solution, one machine, Pareto optimal Solution.

حلول كفوءة لمسألة ترتيب متعددة المقاييس باستخدام الخوارزمية المعدلة

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الخلاصة

أفترضنا في هذا البحث خوارزمية معدلة للتقيد والتفرع (MBAB) لأكتشاف الحلول الكفوءة لمسألة ترتيب متعددة المقاييس على ماكنة واحدة . المقاييس هي وقت الاتمام الكلي ($\sum C_j$) , وقت التأخير الكلي ($\sum L_j$) , وأعظم تأخير لاسالب (T_{max}) .

المجموعة من n المهام المستقلة يجب ترتيبها على الماكنة الواحدة , المهام j تتطلب وقت تشغيل P_j و وقت مثالي d_j. الخوارزمية (MBAB) تعتمد على تقنية التقيد والتفرع . استخدمنا أمثلة تطبيقية لغرض ان نبين قابلية التطبيق لخوارزمية MBAB. تم مقارنة خوارزمية MBAB مع طريقة العد التام (CEM) . الاستنتاجات تمت صياغتها بالاعتماد على الانجاز للخوارزمية (MBAB).

الكلمات المفتاحية : ترتيب متعدد المقاييس , حل كفوء , ماكنة واحدة , حل باريتو أمثل .

Introduction

An optimization problem (OP) is the problem of finding the best solution from all feasible solutions. An OP with discrete variables is known as a combinatorial optimization problem (COP). The multi-criteria MSP, which is considered in this thesis is a (COP) . Machine sequencing problem (MSP) has been an active area of research for the past years. The primary tasks (jobs) in sequencing problem is to determine the sequence in which tasks (jobs) are to be processed on a given set of machines. Tasks (jobs) need to be sequencing on the machines in a way that satisfy the constraints of the problem.

In the literature [1], there are mainly three classes of approaches that are applicable to multi-criteria sequencing problem.

C1: **Hierarchical (lexographical)** optimization the hierarchical approach, one of the criteria (more important) regards as constraint (primary) criterion which must be satisfied, see [2].

C2: **Priority optimization** in this approach minimizing a weighted of the multi-criteria (objectives) and convert the multi-criteria to single criterion problem, several multi-criteria sequencing problems studied in this class, see [3].

C3: **Interactive optimization** In this approach one generates all efficient (pareto optimal) sequencing and select the one that yield the best composite objective function value of the multi-criteria. Several multi-criteria sequencing problems studied in this class, see [4] .

In this paper , the single machine case is involved the jobs j (j=1,2,…,n) request processing time (P_j) due date (d_j) know completion time ($C_j=\sum_{i=1}^{j} P_i$) for particular sequence of job , the lateness criterion $L_j=C_j-d_j$, and the earliness criterion is $E_j=\max\{d_j-C_j,0\}$. Al-Nuaimi [5] proposed the BAB algorithm and presented effective methods for the $1/F(\sum C_j, T_{max}, V_{max})$, $1/F(\sum C_j, \sum V_j, T_{max})$, and $1/F(\sum C_j, T_{max}+V_{max})$ problems to find efficient (Pareto optimal) solutions. In [6] proposed an algorithm to find efficient solution for Multi-criteria problem $1//F(\sum C_j, V_{max}, L_{max})$. In [7] presented an effective algorithm to discover the set of all efficient solution for the multi-criteria problem $1//F(\sum C_j, \sum T_j, L_{max})$. In [8] solved a multi-criteria problem in a hierarchical method. Khamees [9] proposed an efficient (pareto optimal) solutions for the problem $1//F(\sum C_j, \sum L_j, L_{max}, E_{max})$ problem.

Methodology and Materials

Definition (1) (Hoogeveen, 2005)[3]: A feasible sol. (sequence) σ" is efficient(Pareto optimum or non –dominated) with regard to the" performance standard f and g if there is no suitable solution (sequence) π such that both $f(\pi) \le f(\sigma)$ and $g(\pi) \le g(\sigma)$ where on least single(one) of the inequalities is strict.

1. **Branch and Bound method(BAB) [10]**

Is a broad technique for resolving many combinatorial optimization issues. The BAB method is the sequencing solution approach that is employed frequently. This strategy, which can cover an ideal answer (solution) by methodically studying a subset of workable solutions, is a classic illustration of the implicit enumeration methodology. Typically, a search tree with nodes that correspond to this subset is used to describe the operation. A number of new branches emerge from each node of a partially solution, replacing the old one with a set of new, smaller issues (problems) that are mutually incompatible. Two types of branching are frequently employed:

a. Forward branching, in which each job is performed (sequenced) in turn starting at the beginning.

b. Backward branching, in which each job is executed (sequenced) in turn starting at the end.

The BAB approach divides the problem into subsets using a branching procedure and computes bound using a lower bounding procedure in order to minimize an objective function Z for a specific sequencing problem. These techniques are used to eliminate subgroups where no optimal solution has been found. This ultimately yields at least one ideal response (optimal solution). The bounding method is used to determine the lower bound (LB) on each produced sub problem's solution. For each node we determine a (LB), which represents the cost of the sequencing jobs(depending on the the objective function)and the cost of the un sequencing (depending on the determined lower bound)jobs .The upper bound is typically defined as the minimum of all values of recently found feasible solutions. If this node's value (LB) is less than or equal to the upper limit (UB), then branching from this node. If branching reaches a full sequence of tasks, then this sequence is evaluated and if its value LB is less than or equal the current upper limit (UB), the current upper bound (UB) is reset to take that value. The process is repeated until every node in the search tree has been taken into account, i.e, LB>UB for every node. This UB is an ideal answer (optimal solution) to the problem.

2. **Complete enumeration method (CEM)[10]**

Generates sequence one by one, searching for an optimal solution. This method lists all possible sequence and then eliminates the non-optimal sequence from the list, leaving those, which are optimal. Clearly searching for an optimal sequence among all possible sequence using complete enumeration is not suitable even for problems of small size**.**

3. **Formulation of the multi criteria problem:**

The multi criteria sequencing issue problem (P) of total completion time, total lateness and maximum tardiness is formulated as follows:

$$\left.\begin{array}{ll} \text{Min}\{\sum C_j, \sum L_j, T_{max}\} & \\ \text{s.t} & \\ C_j=\sum_{i=1}^{j} P_i & j=1,2,\ldots.n \\ L_j=C_j-d_j\ , & j=1,2,\ldots.n \\ T_j=\max\{C_j-d_j,0\} & j=1,2,\ldots.n \end{array}\right\} \ldots.\ \text{Problem (P)}$$

3.1 Some fundamental and notation concepts of multi criteria sequencing:

N=Set of tasks (jobs)

n=Number of tasks (jobs)

P_j=Processing time for tasks (jobs) j

d_j=Due date for tasks(jobs) j

C_j=completion time for tasks (jobs) j

L_j=lateness for tasks (jobs) j

$\sum C_j$ =total completion time

$\sum L_j$=total lateness

T_{max}=Maximum tardiness

We propose a modified algorithm depends on strategies of branch and bound method and this algorithm is denoted by (MBAB).

3.2 Modified Branch and Bound (MBAB) for finding efficient solutions (Pareto optimal) for the problem 1//F($\sum C_j$,$\sum L_j$, T_{max}) (P):

Step (1): Input date n , p_j and d_j ∀ j=1,2,…..,n .

Step (2): Suppose G=Ø, is the set of efficient solutions, for any sequence σ define F(σ)=($\sum C_{\sigma(j)}$, $\sum L_{\sigma(j)}$,T_{max}(σ)).

Step (3): Set the upper bound (UB) by σ=SPT sequence, where the SPT is sequencing the tasks in non-decreasing order of their processing times. For this order σ, compute F (σ) and the value of UB=$\sum C_{\sigma(j)}$+ $\sum L_{\sigma(j)}$+T_{max} (σ), ∀j=1,2,…,n . Put this UB at the parent node of the search tree.

Step(4): At each node of the search tree of BAB method and for each partial sequence of tasks δ **,** compute a lower bound LB(δ) as follows : LB(δ)= cost of sequencing tasks (δ) , (value of objective function)+ cost of un sequencing tasks obtained by using the two rules SPT for $\sum C_j$ and $\sum L_j$, EDD for T_{max} .

Step (5): Branch from any node with LB≤UB.

Step (6): At the last level of search tree, we get a set of solutions for each application of the definition (2.1), if F(δ) denote the outcome (efficient solution) , then add δ to the set G unless it is dominated by the previously obtained efficient solution in G. Finally we get the set of efficient solutions G.

Step (7): STOP.

The application of MBAB algorithm is clarified by the following examples:

Example (3.2.1): Consider the problem (P) with the following data:

j	1	2	3
P_j	3	2	5
d_j	4	5	7

UB is found by SPT rule as follows:

J	2	1	3
P_j	2	3	5
d_j	5	4	7
c_j	2	5	10
L_j	-3	1	3
T_{max}	0	1	3

$\rightarrow F(\sigma) = (\sum C_{\sigma(j)}, \sum L_{\sigma(j)}, T_{max(\sigma)})=(17,1,3)$

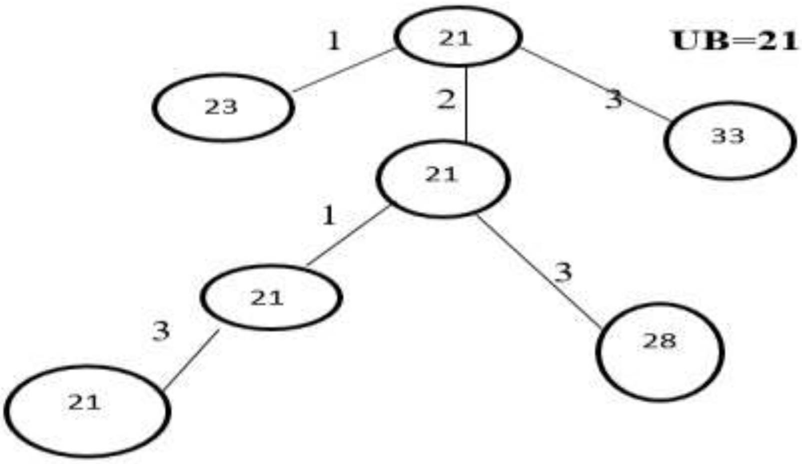
UB $=\sum C_{\sigma(j)} + \sum L_{\sigma(j)} + T_{max(\sigma)} = 21$

In the following table we present the results of applying (MBAB) and (CEM):

Efficient solution for the problem(P)							
Seq.by CEM	$\sum C_j$	$\sum L_j$	T_{max}	Seq. by AIg(MBAB)	$\sum C_j$	$\sum L_j$	T_{max}
(2,1,3)	17	1	3	(2,1,3)	17	1	3

In this example, we show that the efficient sequence by (MBAB) algorithm is equal to that of CEM.

The Modified Branch and Bound Algorithm Tree for example (3.2.1)



Example (3.2.2): consider the problem (P) with the following data:

j	1	2	3	4
p_j	8	7	6	5
d_j	3	4	8	7

UB is found by SPT rule as follows:

J	4	3	2	1
p_j	5	6	7	8
d_j	3	8	4	3
C_j	5	11	18	26
L_j	2	3	14	23
T_{max}	2	3	14	23

$\rightarrow F(\sigma) = (\sum C_{\sigma(j)}, \sum L_{\sigma(j)}, T_{max(\sigma)})=(60,42,23)$

$UB = \sum C_{\sigma(j)} + \sum L_{\sigma(j)} + T_{max(\sigma)} = 125$

In the following table we present the results of applying (MBAB) and (CEM) :

Efficient solution for problem(P)							
Seq. by CEM	$\sum C_j$	$\sum L_j$	T_{max}	Seq. by AIg(MBAB)	$\sum C_j$	$\sum L_j$	T_{max}
(4,3,2,1)	60	38	23	(4,3,2,1)	60	38	23
(4,3,1,2)	61	39	22	4,3,1,2)(	61	39	22
(4,2,1,3)	63	41	18	4,1,2,3)(	64	42	18

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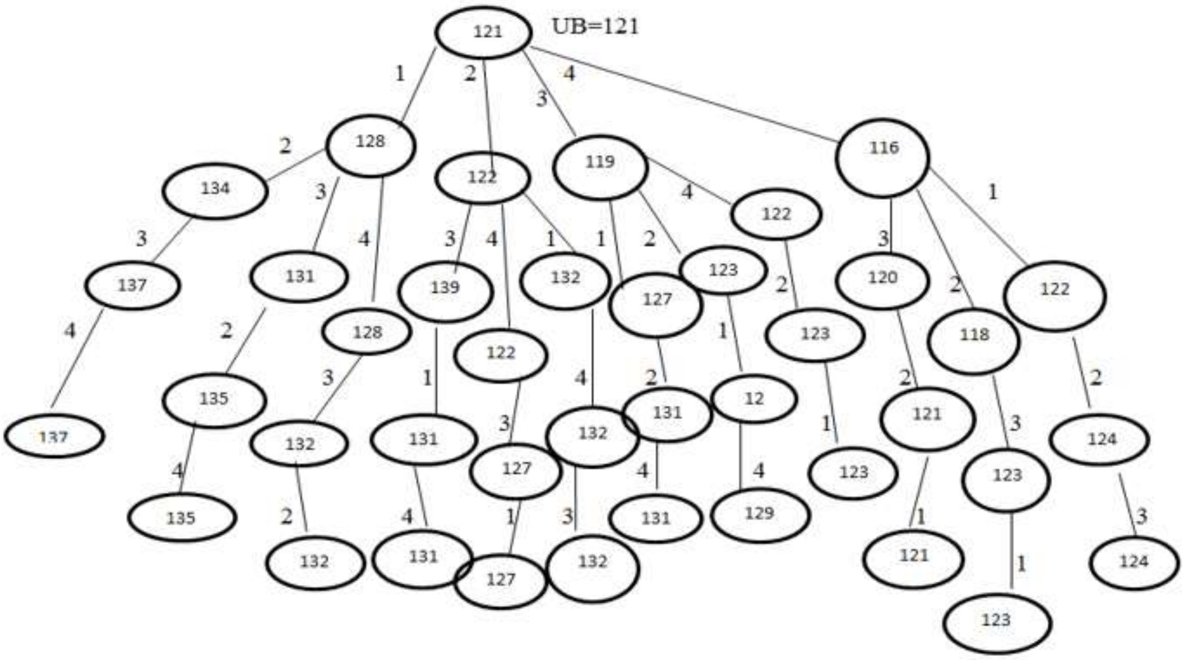
In this example we show that the (MBAB) algorithm give some exact efficient solution and CEM gives all exact efficient solution.

The Modified Branch and Bound Algorithm Tree for example (3.2)



Conclusion

A modified branch and bound algorithm (MBAB) for multi criteria optimization problem is proposed in this paper. The multi- criteria problem is sequencing problem on one machine. (This problem is denoted by P). The algorithm MBAB gives some exact efficient solutions for issue problem (p) effectively. Also, the complete enumeration method (CEM) gives all exact efficient solutions. As a result of our applied examples for (MBAB) algorithm and (CEM), we conclude that the (MBAB) presented have contributes to other multi criteria problems field. We recommend the application of our algorithm (MBAB) in multi- criteria problems in the future research. Also, the experimentation with the one machine multi criteria sequencing problem 1//F($\sum C_j$, $\sum E_j$, L_{max} ,T_{max}).

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