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Applying Some grey prediction models to forecast the number of marriage cases in the city of Halabja, Iraq

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Abstract: Time series forecasting encompasses the examination of historical data to anticipate future values, rely on relevant historical and present data or information for forecasting upcoming values. Thus, gray theory deals with systems with inadequate, poor, and uncertain information. Modeling on insufficient sample and saturation sequences. The objective of this study is to determine the most suitable model from the models proposed (GM (1,1) Model, Discrete Grey Model (DGM) (1,1), Grey Verhulst Model (GVM) (1,1), and Exponential Grey Model (EXGM) (1,1)) for predicting the number of marriages in Halabja governorate, Iraq, in the future. Annual data on the number of marriages from 2016 to 2023 are utilized in this research and Microsoft excel to analyze data. Experimental results indicate that the EXGM (1,1) model is the most accurate model selected in this study, with the lowest average value of MAPE (2.8405%) and a higher level of precision of 97.1595% compared to the other models. This suggests that the EXGM (1,1) model provides more accurate values than the other models. EXGM (1,1) is strongly recommended for forecasting the number of marriages in Halabja governorate, Iraq, for the period 2024-2040.

Keywords: Time series, Grey Model, Mean Absolute Percentage Error.

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تطبيق بعض نماذج التنبؤ الرمادية لتوقع عدد حالات الزواج في مدينة حلبجة، العراق

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المستخلص. شمل التنبؤات الزمنية دراسة البيانات التاريخية للتنبؤ بالقيم المستقبلية، بالاعتماد على البيانات أو المعلومات التاريخية والحالية ذات الصلة لتوقع القيم القادمة. وبالتالي، تتعامل النظرية الرمادية مع الأنظمة التي تعاني من معلومات غير كافية، ضعيفة وغير مؤكدة. النمذجة ضد العينة غير الكافية والتسلسل المشبع. الهدف من هذه الدراسة هو تحديد النموذج الأنسب بين النماذج المقترحة (نموذج GM (1,1))، النموذج الرمادي المتقطع DGM (1,1)، نموذج Verhulst الرمادي GVM (1,1)، والنموذج الرمادي الأسي EXGM (1,1) لتوقع عدد الزيجات في محافظة حلبجة، العراق، في المستقبل. يتم استخدام البيانات السنوية لعدد الزيجات من عام 2016 إلى 2023 في هذا البحث وتحليل البيانات باستخدام Microsoft Excel. تشير النتائج التجريبية إلى أن نموذج EXGM (1,1) هو الأكثر دقة في هذه الدراسة، حيث حقق أقل قيمة لمتوسط النسبة المطلقة للخطأ (MAPE) بنسبة 2.8405% ومعدل دقة أعلى بنسبة 97.1595% مقارنة بالنماذج الأخرى. هذا يشير إلى أن نموذج EXGM (1,1) يوفر قيمًا أكثر دقة من النماذج الأخرى. يُوصى بشدة باستخدام نموذج EXGM (1,1) لتوقع عدد الزيجات في محافظة حلبجة، العراق، للفترة من 2024 إلى 2040.

الكلمات المفتاحية: السلاسل الزمنية، النموذج الرمادي، متوسط النسبة المطلقة للخطأ.

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1. Introduction

Statistical methods widely employ data classification techniques. Forecasting, a tool for predicting future values, relies on pertinent data from both past and present. It encompasses two main approaches: qualitative and quantitative. Qualitative methods, relying on intuition, are typically used in the absence of historical data, while quantitative methods utilize previous data to enable mathematical calculations. Moreover, time series forecasting involves analyzing sequential data to predict future trends or values. Grey theory offers a method for handling systems with limited information or uncertain parameters. Grey models, like GM (1,1), utilize grey theory to forecast by modelling and extrapolating data, particularly in scenarios with incomplete information or uncertainty. Then, the GM(1,1) model and its variations, such as the Discrete Grey Model (DGM(1,1)), Grey Verhulst Model (GVM(1,1)), and Exponential Grey Model (EXGM(1,1)), are effective with very small datasets, often as few as 4 or 5 data points, making them valuable when data is limited or difficult to collect. These models differ in their applications: GM(1,1) uses differential equations to model linear or weakly nonlinear trends, while DGM(1,1) handles discrete, non-continuous data. GVM(1,1) captures systems with growth saturation, like population dynamics, and EXGM(1,1) is ideal for scenarios involving exponential growth or decay. Together, they offer robust forecasting across various data patterns. Additionally, marriage statistics reveal trends in spousal unions, reflecting societal norms and cultural shifts. Analyzing marriage data provides insights into relationship dynamics, family structures, and demographic changes. Understanding marriage patterns helps policymakers, sociologists, and individuals make informed decisions regarding marriage, family planning, and social policies. Marriage data in Halabja governorate-Iraq, reflects local cultural practices and societal norms. Tracking marriage rates provides insight into community dynamics and family structures within the governorate. Halabja governorate located to the east of Sulaymaniyah governorate and nearly 90 KM far from Sulaymaniyah, north of Iraq . The target of this study is to try to find out the most suitable model among four proposed models Grey model GM (1,1), Discrete Grey model DGM (1,1), Grey Verhulst model GVM (1,1) and Exponential Grey model EXGM(1,1) for forecasting the number of marriages in Halabja governorate-Iraq in the future, there are totally 8 observations in the sequence data illustrated in Table (3) which includes yearly data of number of marriages from 2016–2023 which used in this research.

2: Forecasting Models

Grey System Theory, introduced by Deng in 1988, addresses systems lacking complete information, such as structure, operation mechanisms, and behavior documentation, termed Grey Systems. Examples include the human body, agriculture, and the economy. Grey System Theory aims to bridge the gap between social and natural sciences, fostering interdisciplinary collaboration across various fields. It has demonstrated enduring relevance since its inception, particularly evident in China where it's widely known and applied across sectors including agriculture, ecology, economics, meteorology, medicine, geography, and industry.

In Grey System Theory, the GM (m, n) model, where m represents the order of the difference formula and n indicates the number of factors, is employed to predict future system outputs with relatively high accuracy, even without a complete mathematical model of the actual system. The GM (1, 1) model is particularly prominent due to its computational efficiency, with researchers often focusing on it for predictions. This model utilizes first-order differential equations to match data generated by the Accumulation Generating Operation (AGO).

Grey System Theory encompasses five main groups: grey generating, grey relational analysis, grey forecasting, grey decision-making, and grey control. Among these, grey forecasting plays a significant role in addressing uncertainty and information insufficiency within systems. The general form of the grey forecasting model is GM (n, m), where n and m denote the order of ordinary differential equations and the number of grey variables. Various forecasting models are proposed based on different ordinary differential equations and the number of grey variables utilized. The advantage of the GM (1,1) model lies in its computational efficiency, offering accurate predictions despite minimal data requirements. In this study present the concepts of GM (1,1) model, DGM (1,1) model, GVM (1,1) model and EXGM(1,1) model. More specifics are given as follows:

2.1: Grey model

The commonly used grey prediction model is often the GM (1, 1), indicating the utilization of a single variable within the model. The calculation process of GM (1,1) involves six steps, outlined as follows:

Step 1 : The non-negative original sequence data is given by $X^{(0)}$

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}, \quad n \geq 4 \qquad (1)$$

Step 2: Using the non-negative original sequence data $X^{(0)}$, $X^{(1)}$ is constructed by a one-time accumulated generating operation (1-AGO), represented as

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\} \qquad (2)$$

Where:

$$x^{(1)}(1) = x^{(0)}(1)$$
$$X^{(1)}(k) = \sum_{j=1}^{k} x^{(0)} \; (k = 1,2, \dots, n)$$

Step 3: Determine the background value z through mean generating operation (MGO).

$$z^{(1)}(k) = 0.5\, x^{(1)}(k+1) + 0.5\, x^{(1)}(k) \qquad k = 2,3, \qquad (3)$$

Step 4: The result of 1-AGO produces a continuously rising sequence, resembling the solution curve of a first-order linear differential equation. Thus, the solution curve of the following differential equation approximates the 1-AGO data:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \qquad (4)$$

The parameters a and b are referred to as the development coefficient and grey input, respectively. $x^{(1)}(1) = x^{(0)}(1)$represents the corresponding initial condition

Step 5: The parameters a and b can be calculated using the Ordinary Least Squares method (OLS)

$$\binom{a}{b} = (\beta^T \beta)^{-1} \beta^T Y_n \qquad (5)$$

where β and Y_n are defined as follows:

$$\beta = \begin{bmatrix} x^{(1)}(2) & 1 \\ x^{(1)}(3) & 1 \\ \vdots & \dots \\ x^{(1)}(n) & 1 \end{bmatrix} \text{ and } \; Y_n = [x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(n)]^T$$

Solving Eq. (5) along with the initial condition yields the particular solution

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a} \quad k = 1,2,3, \dots \dots \qquad (6)$$

Step 6: Utilizing the inverse accumulated generating operation (I-AGO) on $\hat{x}^{(1)}(k)$, the predicted data of $\hat{x}^{(0)}(k)$ can be estimated as follows:

$$\hat{x}^{(0)}(k+1) = (1 - e^{-a}) \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} \quad (7)$$

Or

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \qquad (8)$$

2.2: Discrete Grey Model

The Discrete Grey Model DGM (1,1) extends Grey System Theory for forecasting with limited data. It utilizes first-order discrete equations, making it suitable for small-sample forecasting. By employing a single-variable model, DGM (1,1) predicts future trends, especially in scenarios with uncertain or incomplete data, the discrete grey model (1,1) was introduced in detail as follows

Step 1 : Assuming the existence of non-negative original sequence data $X^{(0)}$

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}, \quad n \geq 4 \qquad (9)$$

Step 2: We used 1-AGO to construct $X^{(1)}$, aiming to make the sequence $X^{(0)}$, increase by the equation as follows

$$X^{(1)} = \sum_{i=1}^{k} x^{(0)}(1) \quad k = 1,2,3,\dots,n \qquad (10)$$

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(n)\right)$$

Where:

$$X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(1) \quad k = 1,2,3,\dots,n$$

Step 3: The discrete grey model has the form as follows:

$$x^{(1)}(k+1) = ax^{(1)}(k) + b \qquad (11)$$

Where: β_1 and β_2 are model parameters.

Step 4: o calculate the parameters a and b, the ordinary least squares method (OLS) function is utilized as follows

$$\binom{a}{b} = (B^T B)^{-1} B^T Y_n \qquad (12)$$

where B and Y_n are defined as follows:

$$\beta = \begin{bmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \dots \\ x^{(1)}(n-1) & 1 \end{bmatrix} \text{ and } \; Y_n = [x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(n)]^T$$

Particular solution of the discrete grey model is

$$\hat{x}^{(1)}(k+1) = a^k x^{(0)}(1) + \frac{1-a^k}{1-a} \times b \qquad k = 0,1,2, \dots n-1 \qquad (13)$$

where initial condition is $x^{(1)}(1) = x^{(0)}(1)$

Step 5: The predicted datum of $x^{(0)}(k)$ can be estimated by applying the I-AGO $x^{(1)}(k)$ to the following equation:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \qquad k = 1,2,3, \dots \quad (14)$$

2.3: Grey Verhulst Model

Combining Grey System Theory with the Verhulst model, the GVM (1,1) forecasts using first-order discrete equations for small-sample data. The GVM formulation is as follows:

Step 1: Given the non-negative original sequence data $X^{(0)}$

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}, \quad n \geq 4 \qquad (15)$$

Step 2: Utilizing the non-negative original sequence data $X^{(0)}$, $X^{(1)}$ is constructed through the 1-AGO process, represented as

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(n)\right) \qquad (16)$$

$$x^{(1)} = \sum_{i=1}^{k} x^{(0)}(1) \quad k = 1,2,3,\dots,n$$

Where: $X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(1) \quad k = 1,2,3,\dots,n$

Step 3: Calculating a background value z by MGO:

$$z^{(1)}(k) = 0.5x^{(1)}(k+1) + 0.5x^{(1)}(k) \quad k = 2,3,\dots \quad (17)$$

The form of the GVM is as follows:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = bx^{(1)^2}$$

where a is the development coefficient and b is the grey action.

Step 4 : The parameter $\hat{a} = [a,b]^T$ can be handled using the least squares method, employing the function as follows: $\hat{a} = [a,b]^T$ of the GM (1,1) power model is

$$\binom{a}{b} = (\beta^T\beta)^{-1}\beta^T Y_n \quad (18)$$

where B and Y_n are defined as follows:

$$\beta = \begin{bmatrix} -z^{(1)}(2) & (z^{(1)}(2))^2 \\ -z^{(1)}(3) & (z^{(1)}(3))^2 \\ \vdots & \cdots \\ -z^{(1)}(n) & (z^{(1)}(n))^2 \end{bmatrix} \text{ and } \quad Y_n = [x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n)]^T$$

The value taken is $x^{(1)}(0)$, and after solving the mentioned differential equation, the time response equation of the GVM is as follows:

$$\hat{x}^{(1)}(k+1) = \frac{ax^{(1)}(0)}{bx^{(1)}(0) + \left(a - bx^{(1)}(0)\right)e^{ak}} \quad k = 0,1,2,\dots n \quad (19)$$

Step 5 : The resulting grey Verhulst prediction model of X(0)) is as follows

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \quad k = 1,2,3,\dots \quad (20)$$

2.4: Exponential Grey Model

The EXGM(1,1) model introduces a novel approach to grey estimation. It accommodates exponentially varying raw data sequences, where the grey action evolves with time. Unlike the standard GM(1,1), which treats grey action as constant, EXGM(1,1) considers it as an exponential function of time alongside a constant factor (Akyuz and Bilgil 2022).

Step 1 : Given the non-negative original sequence data $X^{(0)}$

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}, \quad n \geq 4 \quad (21)$$

Step 2: Utilizing the non-negative original sequence data $X^{(0)}$, $X^{(1)}$ is constructed by a one-time accumulated generating operation (1-AGO), represented as

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\} \quad (22)$$

Where:

$$x^{(1)}(1) = x^{(0)}(1)$$
$$X^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}\ (k = 1,2,\ldots,n)$$

Step 3: The first-order mean value operator $Z^{(1)}$ is created.

$$Z^{(1)} = \left[z^{(1)}(1), z^{(1)}(2), z^{(1)}(3), \ldots z^{(1)}(n)\right]$$

Where:

$$Z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2} \qquad k = 2,3,\ldots n \qquad (23)$$

Step 4: The governing differential equation of the EXGM (1,1) is

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b + ce^{-1} \qquad (24)$$

The parameters $(a, b\ and\ c)$ can be easily determined by using the least squared procedure

Step 5: Calculating the parameters $(a, b\ and\ c)$ can be calculated using the Ordinary Least Squares method (OLS) functions.

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = (\beta^T\beta)^{-1}\beta^T Y_n \qquad (25)$$

where β and Y_n are defined as follows:

$$\beta = \begin{bmatrix} -z^{(1)}(2) & 1 & (e-1)e^{-2} \\ -z^{(1)}(3) & 1 & (e-1)e^{-3} \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & 1 & (e-1)e^{-n} \end{bmatrix} \text{ and } \quad Y_n = [x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)]^T$$

Here, n denotes the number of samples utilized to build the model. In mathematics, the symbol "e" denotes Euler's number, a fundamental mathematical constant approximately equal to 2.71828

Solving Eq. (25) along with the initial condition yields the particular solution

$$.\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} - \frac{c}{a-1}e^{-1}\right]e^{-a(1-t)} + \frac{b}{a} + \frac{c}{a-1}e^{-t} \quad k, t = 1,2,\ldots \ (26)$$

Step 6: Utilizing the inverse accumulated generating operation (I-AGO) on $\hat{x}^{(1)}(k)$, the predicted datum of $\hat{x}^{(0)}(k)$ can be estimated as follows:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \qquad (27)$$

3: Evaluate Precision of forecasting Models

3.1: Mean Absolute Percentage Error (MAPE)

Several statistical tests and measurements assess the accuracy and performance of the proposed model, among them Mean Absolute Percentage Error (MAPE). To evaluate the reliability and performance of the forecasting technique in this study, the MAPE index was employed. It is defined as follows:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}|\mathrm{PE}_k| * 100\% = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| * 100\% \qquad (28)$$

The forecasting accuracy level can be classified into four grades based on the MAPE of each model as indicated in Table (1):

Table (1) Categorizing the grade of predicting accuracy.

Grade level	Highly accurate	Good	Reasonable	Inaccurate
MAPE	<10%	10% - 20%	20% - 50%	>50%

A lower MAPE indicates higher precision in the forecasting model. Typically, a MAPE below 10% signifies an accurate model, while a MAPE between 10% and 20% denotes a good model with acceptable accuracy

3.2: Precision Rate (p)

Precision Rate, which measures the level of the closeness of the statement of forecast quantity and the actual value, p is defined as follows:

$$p = 100 - MAPE(\%) \quad (29)$$

Table (2) Categorizing the grade of predicting accuracy.

Precision rank	Highly accurate	Good	Reasonable	Inaccurate
Precision rate (p)	$p \geq 99.0\%$	$p \geq 95.0\%$	$p \geq 90.0\%$	$p \leq 90.0\%$

Table (2) shows a higher precision rate indicates greater precision in the forecasting model. Typically, a precision rate greater than 99% signifies an accurate model, while a precision rate between 98.0% and 95.0% suggests a good model with acceptable accuracy.

4. Application

4.1 Introduction:

In this study, time series data regarding the number of marriages in Halabja governorate, Iraq, spanning the years 2016 to 2023, was analyzed. The dataset comprises a total of 8 observations, as outlined in Table (1). The data pertaining to the number of marriages during this period were sourced from the Personal Status Court of the Halabja governorate. The study applies several models to analyze the time series data and forecast the number of marriages. Specifically, the Grey model GM (1,1), Discrete Grey model DGM (1,1), Grey Verhulst model GVM (1,1), and Exponential Grey model EXGM(1,1) are utilized for forecasting the number of marriages in Halabja governorate, Iraq from 2024-2040

4.2 Variable of this Study

The data used for the Grey model GM (1,1), Discrete Grey model DGM (1,1), Grey Verhulst model GVM (1,1), and Exponential Grey model EXGM(1,1) is the number of marriages data from 2016 to 2023. Data can be seen in the table below.

Table (3) shows the number of marriages in Halabja governorate, Iraq, from 2016 to 2023

Years	2016	2017	2018	2019	2020	2021	2022	2023
No. Marriages	726	821	814	935	958	1028	978	990

Source: Personal Status Court of the Halabja governorate.

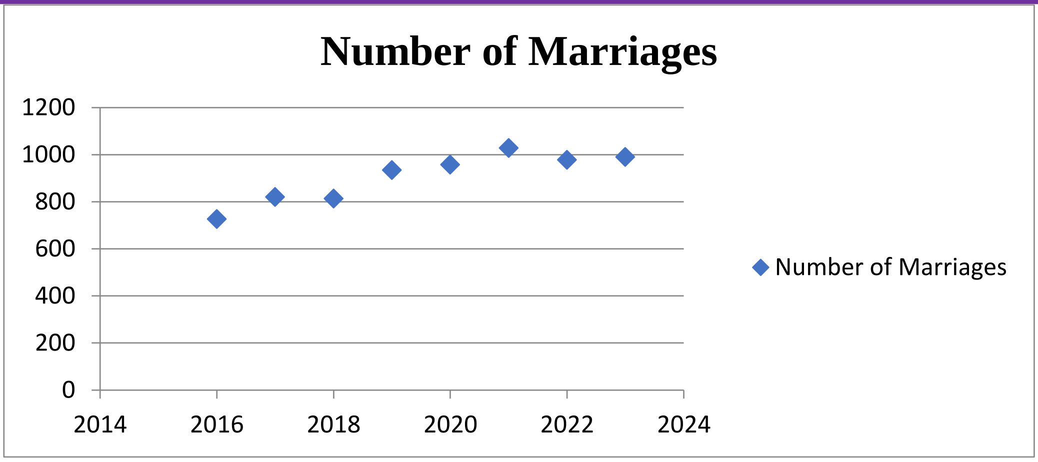

Fig.(1): Plot the number of marriages from (8) years.

Based on Figure 1 of the number of marriages Chart, it can be seen that the data shows obedience in the 2016 period to the 2023 period.

4.3: Results

To determine the most effective model for forecasting the number of marriages from 2016 to 2023 in Halabja governorate, Iraq, this study analyzed the forecasting results to assess variations. Microsoft Excel was utilized for calculating the parameters of the forecasting models. Developed by Microsoft Corporation, this software provides various matrix formulas, including multiple matrices and inverse matrices. In evaluating the accuracy of the forecasting model, the study employed metrics such as the absolute percentage error and the precision rate index.

4.3.1: Estimated Model forecasting of GM (1,1)

Using the Ordinary Least Squares (OLS) method, the parameters of the GM(1,1) model have been estimated as follows: the intercept (a) is -0.034625245, and the slope (b) is 798.5048417.

Table (4) Predicted value and rate error value of the GM (1,1) model.

Year	No. k	Real value	Forecasted value	Percentage Error (PE%)
2016	$k = 0$	726	726.000	0.0000
2017	$k = 1$	821	838.0682	2.0790
2018	$k = 2$	814	867.5947	6.5841
2019	$k = 3$	935	898.1616	3.9399
2020	$k = 4$	958	929.8053	2.9431
2021	$k = 5$	1028	962.5639	6.3654
2022	$k = 6$	978	996.4766	1.8892
2023	$k = 7$	990	1031.5842	4.2004

Source: Prepared by the researchers by obtaining data from (Personal Status Court of the Halabja governorate).

Table above represents the real value , forecasted value and percentage error (PE%) of GM (1,1) model. It is clear seen that the MAPE value for the GM (1,1) is (3.5001%) and it can be said that the efficiency and accuracy values obtained are (p=100−MAPE(%) =100−3.5001% = 96.4999%). which means that the forecasting value is considered good prediction because the value of the precision rate in between 98.0% and 95.0%.

4.3.2: Estimated Model forecasting of DGM (1,1)

The parameters for the DGM(1,1) model have been estimated as follows: the intercept (a) is 1.034988941, and the slope (b) is 813.4074836.

Table (5) Predicted value and rate error value of the DGM (1,1) model.

Year	No. k	Real value	Forecasted value	Percentage Error (PE%)
2016	$k=0$	726	726.0000	0.0000
2017	$k=1$	821	838.8095	2.1692
2018	$k=2$	814	868.1585	6.6534
2019	$k=3$	935	898.5345	3.9001
2020	$k=4$	958	929.9732	2.9256
2021	$k=5$	1028	962.5120	6.3704
2022	$k=6$	978	996.1893	1.8598
2023	$k=7$	990	1031.0449	4.1459

From table (5), it is clear seen that the MAPE value for the DGM (1,1) is (3.5031%) and it can be said that the efficiency and accuracy values obtained are (p=100−MAPE(%) =100− 3.5031% =96.4969%). which means that the forecasting value is considered good prediction because the value of the precision rate in between 98.0% and 95.0%.

4.3.3: Estimated Model forecasting of GVM (1,1)

The parameters For the GVM(1,1) model were obtain as follows $a=-0.485773381$ and $b=-0.000053$ utilizing the least squares solution.

Table (6) Predicted value and rate error value of DGM (1,1) model.

Year	No. k	Real value	Forecasted value	Percentage Error (PE%)
2016	$k=0$	726	726.0000	0.0000
2017	$k=1$	821	398.5501	51.4555
2018	$k=2$	814	573.4895	29.5467
2019	$k=3$	935	776.3200	16.9711
2020	$k=4$	958	968.3261	1.0779
2021	$k=5$	1028	1091.7175	6.1982
2022	$k=6$	978	1099.0545	12.3778
2023	$k=7$	990	987.2705	0.2757

Source: Prepared by the researchers by obtaining data from (Personal Status Court of the Halabja governorate).

From table (6), it is clear seen that the MAPE value for the DGM (1,1) is (14.7379%) and it can be said that the efficiency and accuracy values obtained are (p=100−MAPE(%) =100−14.7379%=85.2621%). which means that the forecasting value is considered good prediction because the value of the precision rate in between 98.0% and 95.0%.

4.3.4: Estimated Model forecasting of EXGM (1,1)

we obtain the parameters of the EXGM(1,1) model using the least squares procedure as follow:
$a=-0.021342903,\quad b=870.9680117\quad$ and $c=-404.7903669$

Table (7) Predicted value and rate error value of DGM (1,1) model.

Year	No. k	Real value	Forecasted value	Percentage Error (PE%)
2016	$k=0$	726	726.0000	0.0000
2017	$k=1$	821	800.6800	2.4750
2018	$k=2$	814	878.1998	7.8870
2019	$k=3$	935	919.3083	1.6783
2020	$k=4$	958	947.2935	1.1176
2021	$k=5$	1028	970.7283	5.5712
2022	$k=6$	978	992.7726	1.5105
2023	$k=7$	990	1014.5949	2.4843

From table (7), it is clear seen that the MAPE value for the DGM (1,1) is (2.8405%) and it can be said that the efficiency and accuracy values obtained are (p=100−MAPE(%)

=100−2.8405%=97.1595%). which means that the forecasting value is considered good prediction because the value of the precision rate in between 98.0% and 95.0%.

Table (8) Represents the accuracy of models

Criteria	MAPE (%)	Precision Rate (p)
GM(1,1)	3.5001	96.4999
DGM(1,1)	3.5031	96.4969
GVM(1,1)	14.7379	85.2621
EXGM(1,1)	2.8405	97.1595

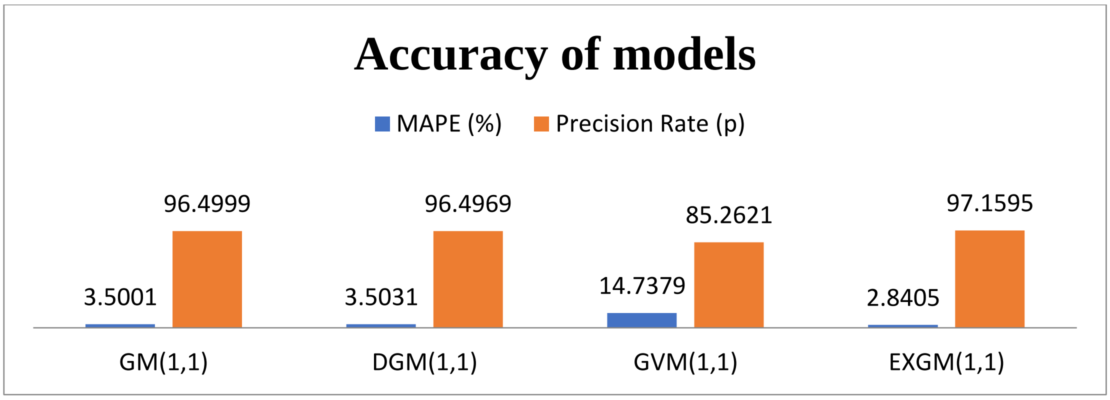

Fig.(2): Plot the accuracy of models

In Table (8), it can be observed that the mean absolute percentage error (MAPE%) and precision rate (p) are provided for the GM (1,1), DGM(1,1), GVM(1,1), and EXGM(1,1) models. The MAPE value for the EXGM(1,1) model is 2.8405%. Consequently, the effectiveness and accuracy of the EXGM(1,1) model surpass those of the GM (1,1), DGM(1,1), and GVM(1,1) models, as the MAPE value of the EXGM(1,1) model is lower compared to the others. Additionally, the precision rate (p) of the EXGM(1,1) model is higher at 97.1595% when compared to the other models. Therefore, based on these results, this study recommends the utilization of the EXGM(1,1) model for future estimations of the number of marriages in Halabja governorate-Iraq. The forecasted values for 2024 to 2040 are presented in Table (9).

Table (9) The forecasted value by EXGM(1,1) model

Years	2024	2025	2026	2027	2028
No. Marriages	1037	1059	1082	1105	1129
Years	2030	2031	2032	2033	2034
No. Marriages	1178	1204	1230	1256	1283
Years	2036	2037	2038	2039	2040
No. Marriages	1339	1368	1398	1428	1459

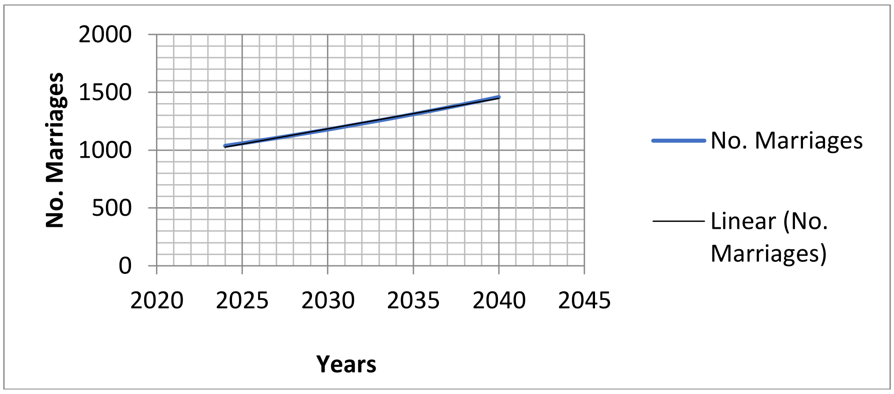

Fig.(3): Plot the forecasted No. Marriages by EXGM(1,1) model

5. Discussion

This study presents a comparison between the GM (1,1), DGM(1,1), GVM(1,1), and EXGM(1,1) models for forecasting the number of marriages in Halabja governorate, Iraq, over the period of 2024 to 2040. The comparison is based on two criteria: mean absolute percentage error (MAPE) and precision rate (p), aimed at determining the most effective method for forecasting the number of marriages in Halabja governorate, Iraq. A lower MAPE and a higher precision rate indicate that a model can achieve greater accuracy in predicting future values. Typically, a precision rate exceeding 99% indicates a highly accurate model, while rates between 98.0% and 95.0% are considered acceptable with good accuracy. The most appropriate model for predicting the number of marriages in Halabja governorate, Iraq, is the EXGM (1,1) model. This conclusion is based on the fact that the EXGM (1,1) model has the lowest values of mean absolute percentage error (MAPE) and the highest Precision Rate (p) compared to other models, with percentages of 2.8405% and 97.1595%, respectively. A lower value of the accuracy measurement indicator signifies a more reliable model, making it an appropriate analysis tool for forecasting. The EXGM (1,1) model is selected as the most accurate method when compared to other models in this study. Therefore, in the analysis of the number of marriages in Halabja governorate, Iraq, the EXGM (1,1) method emerges as the best approach.

6. Conclusion and Recommendation

In this paper, we compare the accuracy of the grey forecasting models to predict the number of marriages in Halabja governorate-Iraq. The grey system theory could deal with the problems with incomplete or unknown information and also the small sample, so this paper uses it. In this paper, four grey forecasting models named as GM (1,1), DGM (1,1), GVM and EXGM(1,1) models were used in order to analyze the efficiency of model forecasting for the number of marriages in Halabja governorate-Iraq. Furthermore, this study determined that EXGM (1,1) is the most effective model for forecasting the number of marriages depends on the mean absolute percentage error (MAPE) and precision rates (p). The direct implication of this study is to offer a valuable tool for decision-makers in constructing models to forecast the number of marriages. The study suggests that future research endeavors should explore additional grey forecasting models to enhance predictive accuracy. Specifically, it recommends applying and comparing the Grey model and Fourier Grey model in future studies

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