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Solving Multi-Criteria Problem by Using Efficient Algorithm

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Abstract

For multi criteria sequencing problem on one machine, we propose an algorithm total completion total lateness maximum lateness maximum earliness (CLLE) to find efficient solutions in this paper. The criteria are total completion time($\sum C_j$), total lateness($\sum L_j$), maximum Lateness (L_{max}) and maximum earliness (E_{max}). A collection of n independent jobs (tasks) has to be sequenced on one machine jobs j (j=1, 2, 3, …., n) wanted processing time pj and due date dj. Conclusions is formulated on the competence of the (CLLE) algorithm based on results from computational experiments.

keywords: Multi criteria, Sequencing, one machine, Efficient Solution, Algorithm.

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حل مسألة متعددة الأهداف باستخدام خوارزمية كفؤة

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الخلاصة

اقترحنا خوارزمية (CLLE)لايجاد حلول كفؤة لمسألة ترتيب متعددة الأهداف في هذا البحث. الأهداف هي زمن الاتمام الكلي ($\sum C_j$)، زمن التأخير الكلي ($\sum L_j$)، أعظم تأخير (L_{max}) وأعظم تبكير (E_{max}) تجمع من n من المهام المستقلة التي يجب ترتيبها على الماكنة الواحدة، المهام j($j=1,2,\ldots,n$) تتطلب وقت تشغيل P_j و وقت مثالي d_j. بالاعتماد على نتائج التجارب الحسابية تم صياغة الاستنتاجات على كفاءة الخوارزمية (CLLE).

الكلمات المفتاحية: ترتيب، خوارزمية، متعددة الهدف، ماكنة واحدة، حل كفوء.

Introduction

There has been a large amount research on output sequencing problems since the original of mathematical formulations typically, this entails assigning machines to process tasks (jobs) over time in order to refine certain output parameters, either precisely or roughly.

The literature can be divided into two major categories [1]:

a) Deterministic sequencing research: where all problem parameters are to be well-known.

b) Stochastic sequencing research: where at least some parameters are random variables.

On deterministic sequencing research a large view is taken and multiple machines are often modeled. The deterministic approach is to plan the work out of the machines over a period of time in the best possible way, given a specific criterion to optimize. The implicit assumption here is often that a sequence can be executed directly you develop.

However, several scholars have recently recognized that this unlikely scenario exists in many manufacturing settings, and have attempted to apply the deterministic approach to circumstances involving various complexity [1].

In this study, the single machine case is considered the jobs j (j=1, 2, 3, .., n)request processing times (p_j) due date (d_j), completion times ($C_j=\sum^{j}_{i=1}p_i$) for particular sequence of job .the lateness criterion $L_j=C_j-d_j$ and the earliness criterion is $E_j=\max\{d_j-C_j,0\}$[2][2014]. [3] [2018] presented an effective algorithm to discovery the set of all efficient solutions for the multi-criteria problem $1//f(\sum C_j, \sum T_j, L_{max})$ [4][2019]. Proposed an algorithm to find efficient solutions for Multi-criteria problem $1//F(\sum C_j, V_{max}, L_{max})$. [4] [2019]. Reduced Multi-criteria scheduling problems to bi-criteria sequencing problems [5] [2021].[6] [2022] solving a multi-criteria problem in a hierarchical method.

Basic Concepts of Machine Sequencing Machine

Definition Hoogeveen: A feasible solution (sequence) σ is efficient (Pareto optimal or non – dominated) with regard to the performance criteria f and g if there is no feasible solution (sequence) π Such that both $f(\pi) \leq f(\sigma)$ and $g(\pi) \leq g(\sigma)$ where at least one of the inequalities is strict [7] (2005). Lawler's Algorithm (LA) which solves the $1/pres/f_{max}$ problem or $1//f_{max} \in (C_{max}, L_{max}, T_{max}, V_{max})$ [8] [2008] to find minimum f_{max} Lawler's Algorithm (LA) is described by the following steps (algorithm 1):

Algorithm; Lawler's algorithm:

Step (1): let N = {1, 2, …., n} F is the set of all tasks (jobs) with no successor and $\pi=\phi$

Step (2): let J* be a job such that $f_{j^*}(\sum_{j\in N} p_i) = \min_{i\in f}\{f_j(\sum_{i\in N} p_i)\}$

Step (3): Set N=N-{J*} and sequence job J* in last position of π, i.e., π = (π, J*).

Step (4): Modify F with respect to the new set of sequencing jobs

Step (5): LF $N=\phi$ stop, otherwise go to step (2).

Formulation of the simultaneous multi criteria problem

The simultaneous multi criteria sequencing (P) problem of total completion time, total lateness maximum lateness and maximum earliness is formulated as follows [8]:

$$\left.\begin{array}{l} \text{Min } \{\sum C_j, \sum L_j, L_{max}, E_{max}\} \\ \text{s.t} \\ C_j = \sum_{i=1}^{j} P_i \qquad j = 1, 2, ..., n. \\ L_j = C_j - d_j, \quad j = 1, 2, \ldots n. \\ E_j = \max\{d_j - c_j, 0\}, \quad j = 1, 2, \ldots, n \end{array}\right\} \qquad \ldots P$$

3.1 Notation and some fundamental concepts of multi criteria sequencing:

N= Set of jobs

n=Number of jobs

P_j= processing time for jobs j

d_j=Due date for job j

C_J= completion time for jobs j

L_j= lateness for jobs j

L_{max}=Maximum lateness

E_{max} = Maximum earliness

$\sum C_{j=}$ total completion time

$\sum L_j$= total lateness

SPT= Shortest processing time

MST=Minimum slack time

An algorithm (CLLE) to find efficient solution of the (p) problem

1//F ($\sum C_j$, $\sum L_j$, L_{max}, E_{max}) (p):

Step (1): Let $\Delta=\sum p_i$, $\sigma = (\phi)$.

Step (2): Let N={1,2,…,n} k=n, $t=\sum p_i$.

Step (3): Calculate L_i by Lawler 's algorithm ($\forall_i \in N$).

Step (4): Find $j\in N \ni L_j \leq \Delta$, $P_j \geq P_i$, $\forall$ j, i $\in$ N & $L_i \leq \Delta$, assign j in position k of σ, If no job j with $L_j \leq \Delta$, set $E_{max}(\sigma) = E_{max}(SPT)$ go to step (8).

Step (5): Set $t=t-p_j$, N=N-{j}, k=k-1, if k >1 go to step (3).

Step (6): For the resulting sequence job σ = (σ(1),…,σ(n)), calculate ($\sum C_i(\sigma)$, $\sum L_i(\sigma)$,$L_{max}(\sigma)$, $E_{max}(\sigma)$).

Step (7): Put $\Delta = L_{max}(\sigma)-1$, go to step (3).

Step (8): Put $\Delta = E_{max}(\sigma)-1$ N={1,…,n}, k=1, $t=\sum p_i$ and σ=(ϕ) if $\Delta \leq E_{max}$ (MST) go To step (12).

Step (9): Calculate $r_i = \max\{S_i-\Delta,0\}$ $\forall_i \in N$.

Step (10): Find a job j∈ N with $r_j \leq C_{k-1}$ and $P_j \leq P_i$ ∀ j, i ϵ N , $C_0 = 0$ (break tie with min S_j) assign j in position k of σ .

Step (11): Set N=N-{j}, k=k+1 , if k ≤ n go to step (10) for the resulting Sequence σ = (σ(1),…,σ(n)) calculate ($\sum C_i(\sigma)$, $\sum L_i(\sigma)$, $L_{max}(\sigma)$, $E_{max}(\sigma)$) And go to step (8).

Step (12): Stop with a set of efficient solutions.

Test problems with computational experiments

The (CLLE) algorithm is tested for (P) problem for generating efficient solution through coding it in MATLAB 2019 and running on a personal computer HP at Ram 2.50 GB test Problems are produced as follows :for each job j an integer processing time P_j produced From the discrete uniform distribution [2][2014].Also ,to every job j an integer due date d_j is generated from the discrete regular distribution [P(1-TF-RDD/2) ,P(1-TF+RDD/2)] where, $P=\sum^{p}_{i=1} p_i$ depening. On the relative range of due date (RDD) and on the average tardiness factor (TF). For our problem the experimental results on a set of test problems for the complete enumeration method (CEM) solve the problems of size n= 3,4,…,8. The (CEM) consumes much time, because it seeks all solutions while the (CLLE) algorithm takes less time. For both parameters, the values 0.2,0.4,0.6,0.8,1.0 are considered. For selected value of n , two problems are generated for each of the five number of parameters making 10 problems for each account of n where the number of jobs n ≤ 1000 for the (P) problem .Average calculation time in seconds and average time in one more and average number of efficient points for algorithm (CLLE) algorithm for n= 9,…,1000 are given in table (1).

Table 1: Average calculation time in second and rate no. of efficient Points for CLLE algorithm.

NO .OF JOBS (N)	AVERAGE COMPUTATION TIME	AVERAGE NUMBER OF EFFICIENT POINTS
9	0.019326	8.6
10	0.0024144	7.6
11	0.0027417	8.6
12	0.0034076	11.6
13	0.004011	1.4
14	0.00667603	18.5
15	0.0073207	2.2
16	0.010693	25
17	0.0132587	26.7
18	0.0127125	24.1
19	0.0132143	22.8
20	0.0143732	22
21	0.0171874	26.5
22	0.0206854	28,5
23	0.0379726	3.5
24	0.0286329	31.4
25	0.0325804	37.3
26	0.0373847	4.3
27	0.03809491	37.2
28	0.04821883	53.4
29	0.0482349	42.2
30	0.0686457	57.4
31	0.06253185	55.4
32	0.0748663	55.7
33	0.076874	6.2
34	0.0758339	48.8
35	0.0888483	55.1
36	0.1219888	74.4
37	0.1131517	75.6
38	0.1243789	68
39	0.1130938	57.9
40	0.15582349	76.6
41	0.1634846	8.8
42	0.2096598	98.1
43	0.175568	76.7
44	0.207294	87.9
45	0.2107595	93.9
46	0.2491186	91.9
47	0.25033966	72.1

48	0.3307062	114.3
49	0.2472714	88.4
50	3.31712011	13.6
75	0.9944874	166.6
100	2.3576311	245.3
125	5.25529296	34.4
150	9.1481934	386.1
200	10.5642111	493
250	18.44231389	617.2
300	1.02659037618	803.9
350	50.4026364	933.1
400	87.89174984	136.6
450	113.834893	1335.6
500	141.25137681	138.5
750	532.5846138	2249.2
1000	883.97406831	2634.4

Conclusion and future work

In this study, an algorithm (CLLE) is presented to solve multi criteria optimization and investigated its performance on a specific single machine multi criteria sequencing problem(P). Since we are using (CLLE) algorithm, we can be sure that a solution for problem(P)is truly an efficient solution. As a result of our experiments, we conclude that the (CLLE)algorithm performs quite well for the multi criteria problem (P)of size n=9,10,…,1000. the research presented, here, contributes to the multi criteria sequencing literature by adapting (CLLE) algorithm to multi criteria problems. For future research, we recommend the topic that would use experimentation with the following machine sequencing problem.

$1//F(\sum C_j, \sum E_j, \sum L_j, L_{max})$

$1//F(\sum C_j, \sum L_j, \sum T_j, E_{max})$

$1//F(\sum C_j, \sum E_j, \sum T_j, \sum L_j, L_{max}, E_{max})$.

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