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Transforming and Solving Multi-Objective Linear Plus Linear Fractional Programming Problem

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Abstract

Multi-objective optimization also known as multi-objective programming is an area of multiple criteria decision making that is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously. In such circumstance, we have to be discovered out compromise arrangement which is ideal for all the objectives in a few senses. In this paper, we transformed multi-objective linear plus linear fractional programming problems to single QPP and then solved by methods of QPP. Illustrative numerical examples are displayed for exhibit reason. We have explored an arrangement to the MOLPLFP issue based on a hypothesis already considered by Dinkelbach. He clearly delineated a calculation for fractional programming with nonlinear as well as linear terms within numerator and denominator.

Keywords: Fractional linear programming, Multi-objective linear programming, Linear programming, Quadratic programming problem.

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البرمجة الخطية متعددة الاهداف الخطية مع مشكلة البرمجة الكسرية الخطية

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الخلاصة

تعد مسالة ثنائية الاهداف احدى مسائل اتخاذ القرار للمشاكل التي تحتاج الى حل مسائل تحتوي على اكثر من دالة هدف واحدة انيا هذه المسائل تتطلب حلا وسيطا لجميع الاهداف الموجودة. في هذا البحث حولنا مشكلة دالة خطية متعددة الأهداف مع دالة كسرية خطية الى مسالة البرمجة التربيعية ومن ثم حلها بطرق البرمجة التربيعية المعروفة تم اعطاء امثلة توضيحية لهذة المسائل بالاعتماد على نظرية (دينكلباغ) والذي يوضح فيها الخوارزمية للبرمجة الكسرية.

الكلمات المفتاحية: البرمجة الكسرية الخطية، البرمجة الخطية متعددة الاهداف، البرمجة الخطية، البرمجة التربيعية.

Introduction

Linear fractional Programming (LFP) could be a generalization of linear programming (LP) while the objective function in a linear program is a linear function; the objective function in a linear-fractional program is a ratio of two linear functions [1]. The fractional programming problems are especially valuable within arrangement of financial issues in which various activities utilize certain resources in various proportions, while the goal is to optimize a certain pointer, ordinarily the foremost favorable return on allotment proportion subject to the certain forced on the availableness of goods [2].

Subsequently, (MOLPLFPP) comprises of different goals be the mix of straight and straight fragmentary programming. Charnes with Cooper [3] substituted any linear fractional programming problems with, at most, two straightforward linear programming problems that contrast from each other by only a change in sign in the functional and in same constraint. Afterward, Zoints [4] appeared that in case a limited arrangement to the issue exists, only one direct programming issue must be solved. Schaible [5] studied onto the whole of a straight and linear-fractional function. Bitran with Novaes [6] displayed a modern approach for solving linear fractional programming problem by changed it into linear programming (LP) issue and thus solve this issue logarithmically utilizing the concept of duality. In 1981, Kornbluth and Steuer [7] presented objective function procedures to resolve multi objective linear fractional programming problem by alter strategy. In 1982, Choo and Atkins [8] gave a careful examination of the bi-criteria case in linear fractional programming problem. Hirche [9] provided a note of (LPLFP). Nykowski with Zolkiewski [10], also Dutta with Tiwari. [11], Chadha [12], Chakraborty and Gupta [13], Pal and Moitra together with Maulik [14], Guzel with Sivri [15], moreover studied in linear fractional. In 2008, Mangal and Sanjay together with Parihar [16] displayed a strategy to solve (MOLPLFPP) includes non-differentiable term within limitations. In addition, Kheirfan [17] recommended a method to affectability examination for (LPLFPP). Sharma and Kumar [18] illuminated linear plus linear fractional interval programming problem.

In this study, we transformed and solved (MOLPLFPP) to QPP where its arrangement method can be effortlessly connected.

Formulation of (MOLPLFP) Problems

The general form of (MOLPLFPP) as follows:

$$\left.\begin{aligned} &Max.\, Z_i(x) = \left(C_i^T x + d_i\right) + \frac{\gamma_i^T x + \alpha_i}{\delta_i^T x + \beta_i} \\ &subject\ to\!: x \in S = \{x \mid Ax \le = \ge b, x \ge 0\} \end{aligned}\right\} \quad (1)$$

Where, $C_i^T, \gamma_i^T, \delta_i^T \in R^n, i = 1, 2, \dots, k$ and $A \in R^{n \times m}, b \in R^m, S$ is anticipated to be non-empty, arched and compact belong to R^n, $(\delta_i^T x + \beta_i) > 0$ for $i = 1, 2, \dots, k$.

In our work, we study the objective function such as in a problem (1) is a composite of two terms. The first term is straight and the second term is fragmentary with the linear numerator and denominator.

The Relationship between Nonlinear Fractional and Nonlinear Parametric Programming

Let E^n be the Euclidean space of dimension n and S be a compact and associated subset of E^n. Let $P(x)$, $N(x)$and $D(x)$ be continuous and real-valued functions of $x \in S$. Furthermore, the following assumption is also made:

$D(x) > 0 \;\; \forall \; x \in S$. We interested by the next two problems:

$$\boldsymbol{max} \, \{\boldsymbol{P}(\boldsymbol{x}) + \boldsymbol{N}(\boldsymbol{x}) \backslash \boldsymbol{D}(\boldsymbol{x}) \mid \boldsymbol{x} \in \boldsymbol{S}\} \quad \textbf{(2)}$$

$$\boldsymbol{max} \, \{\boldsymbol{P}(\boldsymbol{x})\boldsymbol{D}(\boldsymbol{x}) + \boldsymbol{N}(\boldsymbol{x}) - \boldsymbol{z}^* \boldsymbol{D}(\boldsymbol{x}) \mid \boldsymbol{x} \in \boldsymbol{S}\} \quad \textbf{(3)}$$

The problems (2) and (3) have solutions, indeed, and the singular points defined by D(x) = 0 are avoided.

Theorem:

$z^* = P(x_0) + N(x_0) \backslash D(x_0) = max\{P(x) + N(x) \backslash D(x) \mid x \in S\}$ if and only if

$F(z^*) = F(z^*, x_0) = \max\{P(x)D(x) + N(x) - z^* D(x) \mid x \in S\} = 0.$

Proof: Let x_0 be a solution of problem (2), we have;

$z^* \geq P(x) + N(x)\backslash D(x) \rightarrow z^* - P(x) \geq N(x)\backslash D(x) \rightarrow z^* D(x) - P(x)D(x) \geq N(x).$

Then, $\boldsymbol{P(x)D(x) + N(x) - z^* D(x) \leq 0}$ **for all** $\boldsymbol{x \in S}$ **(4)**

$z^* = P(x_0) + N(x_0)\backslash D(x_0) \rightarrow z^* - P(x_0) = N(x_0)\backslash D(x_0) \rightarrow z^* D(x_0) - P(x_0)D(x_0) = N(x_0)$

Then, $\boldsymbol{P(x_0)D(x_0) + N(x_0) - z^* D(x_0) = 0}$ **(5)**

From equation (4) we have

$$F(z^*) = \max\{P(x)D(x) + N(x) - z^* D(x) \mid x \in S\} = 0$$

From equation (5).

$$P(x_0)D(x_0) + N(x_0) - z^* D(x_0) = 0$$

That is, $F(z^*, x_0) = \{P(x_0)D(x_0) + N(x_0) - z^* D(x_0)\} = 0.$

In the other side, let x_0 be a solution of problem (3), we have;

$$P(x_0)D(x_0) + N(x_0) - z^* D(x_0) = 0.$$

The definition of (3) implies

$P(x)D(x) + N(x) - z^* D(x) \leq P(x_0)D(x_0) + N(x_0) - z^* D(x_0) = 0$ for all $x \in S$.

Hence,

$$\boldsymbol{P(x)D(x) + N(x) - z^*D(x) \leq 0, \text{ for all } x \in S,} \quad (6)$$

$$\boldsymbol{P(x_0)D(x_0) + N(x_0) - z^*D(x_0) = 0,} \quad (7)$$

From equation (6).

$P(x)D(x) + N(x) - z^*D(x) \leq 0 \rightarrow z^*D(x) \geq P(x)D(x) + N(x) \rightarrow z^* \geq P(x) + N(x)\backslash D(x)$ for all $x \in S$.

That is z^* is a maximum of problem (2). From equation (7).

$z^* = P(x_0) + N(x_0)\backslash D(x_0)$ that is x_0 is also vector of (2).

Proposed Approach for MOL Plus LFP problems

The maximum of (MOLPLFPP) is characterized as:

$$\textbf{Maximize}\begin{Bmatrix} \boldsymbol{Z(x) = \left(P_1(x) + \frac{N_1(x)}{D_1(x)}, P_2(x) + \frac{N_2(x)}{D_2(x)}, \dots, P_k(x) + \frac{N_k(x)}{D_k(x)}\right) x \in S} \\ \boldsymbol{Ax \leq b, x \geq 0} \end{Bmatrix} \quad (8)$$

Where, A is $m \times n$ constraint matrix, x is an n −dimensional vector of decision variable, and $b \in R^m$, $k \geq 2$, $P_i(x) = a_i^T x + \delta_i$, $N_i(x) = c_i^T x + \alpha_i$, $D_i(x) = d_i^T x + \beta_i$, $a_i^T c_i^T, d_i^T \in R^n$, $\delta_i, \alpha_i, \beta_i \in R$, $D_i(x) = d_i^T x + \beta_i > 0$, for all $i = 1,2, \dots, k$.

In this work, in arrange to unravel issue (8), we are solving each objective function $Zi(x)$ subject to the given set of constraints utilizing one of the strategies proposed for Linear programming and single fractional objective function in [19] or others. Let Z_i^* be the values of each objective function Max$\{Zi(x) = (a_i^T x + \delta_i) + (c_i^T x + \alpha_i)/(d_i^T x + \beta_i) | x \in X\}$ at x_i^* which is the global maximum points for all $i = 1, 2, . . . , k$. Presently, we can prove that the solution $\bar{x}$ is an efficient solution of Max{ $Zi(x) = (a_i^T x + \delta_i) + (c_i^T x + \alpha_i)/(d_i^T x + \beta_i)$, i=1,2,…,k | $x \in X$}.

If $\bar{x}$ is an optimal solution of problem $Max\{\sum_{i=1}^{k}(P_i(x)D_i(x) +, N_i(x) - Z_i^* D_i(x)$ | x ∈ S}, where is $Z_i^* = P_i(x_i^*) +, N_i(x_i^*)/D_i(x_i^*) \quad \forall i = 1,2, \ldots, k.$

Let $\bar{x}$ maximise problem $Max\{\sum_{i=1}^{k}(P_i(x)D_i(x) + N_i(x) - Z_i^* D_i(x)$ | x ∈ S}; then we can write inequality $\sum_{i=1}^{k}(P_i(x)D_i(x) +, N_i(x) - Z_i^* D_i(x)) \le \sum_{i=1}^{k}\big(\big(P_i(\bar{x})D_i(\bar{x}) +, N_i(\bar{x}) - Z_i^* D_i(\bar{x})\big)$ for any feasible solution x∈S. Hence,

$$\sum_{i=1}^{k}(P_i(x)D_i(x) + N_i(x) - Z_i^* D_i(x)) \le \sum_{i=1}^{k} \big(\big(P_i(\bar{x})D_i(\bar{x}) + N_i(\bar{x}) - Z_i^* D_i(\bar{x})\big).$$

$$\le \sum_{i=1}^{k} max(P_i(x)D_i(x) + N_i(x) - Z_i^* D_i(x)),$$

$$\le \sum_{i=1}^{k} P_i(x_i^*)D_i(x_i^*) + N_i(x_i^*) - Z_i^* D_i(x_i^*) = 0 \quad \text{for} x \in X.$$

From these inequalities, one obtains $P_i(x)D_i(x) + N_i(x) - Z_i^* D_i(x) \le P_i(\bar{x})D_i(\bar{x}) + N_i(\bar{x}) - Z_i^* D_i(\bar{x}) \le 0$, for all $i, x \in X$.

Proposed Algorithms

We transform (MOLPLFPP) to QPP, construct to a hypothesis previously examined by Dinkelbach [20], then we can solve QPP easily.

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Numerical Examples

1. Consider the MOLPLFP Problem

$$Max.Z_1 = (x_1 - 2) + \frac{x_1}{(x_1+x_2)}$$

$$Max.Z_2 = (x_1 - 4) + \frac{x_2}{(x_2-2)}$$

Subject to: $x_1 + 3x_2 \leq 9$

$$x_1 + 5x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

Solution:

The best and worst solutions for each objective function are getting as follows:

$$-2 \leq Z_1 \leq 4 , \quad -5 \leq Z_2 \leq 1$$

This (MOLPLFPP) is identical to the following QPP. The given (MOLPLFP) issues can be composed as follows:

$$\mathrm{Max.\,Z} = \{x_1^2 + 2x_1x_2 - 7x_1 - 10x_2 + 10\},$$

solve this QPP with the same constraints we get

$\mathrm{Max.\,Z} = 10$ at (0,0).

2. Consider the MOLPLFP Problem

$$Max. Z_1 = (-x_1 - 1) + \frac{-5x_1+4x_2}{2x_1+x_2+5}$$

$$Max. Z_2 = (x_2 + 1) + \frac{9x_1+2x_2}{7x_1+3x_2+1}$$

$$Max. Z_3 = (x_1 + 1) + \frac{3x_1+8x_2}{4x_1+5x_2+3}$$

Subject to: $4x_1 + 5x_2 \leq 25$

$x_1 + 9x_2 \geq 9$

$x_1, x_2 \geq 0.$

Solution:

The best and worst solutions for each objective function are getting as follows:

$-8.4 \leq Z_1 \leq -7.3$, $2.6 \leq Z_2 \leq 3.2$, $6.7 \leq Z_3 \leq 7.5$

This (MOLPLFPP) is identical to the following QPP. The given MOLPLFP problem can be composed as follows:

$$Max. Z = \{2x_1^2 + 11x_1x_2 + 3x_2^2 - 23.8x_1 - 17.8x_2 + 9.8\}$$

Solve this QPP with the same constraints we get:

$$Max. Z = -19\ at\ (5,1).$$

3. Consider the MOLPLFP Problem

$$Max.Z_1 = (-x_1 - 1) + \frac{(-x_1+2x_2-5)}{(7x_1+3x_2+1)}$$

$$Max.Z_2 = (-2x_2 - 1) + \frac{(2x_1-3x_2-5)}{(x_1+1)}$$

$$Max.Z_3 = (-3x_1 - 1) + \frac{(5x_1+2x_2-19)}{(5x_1+20)}$$

Subject to: $x_2 \leq 6$

$$2x_1 + x_2 \leq 9$$

$$-2x_1 + x_2 \leq 5$$

$$x_1 - x_2 \leq 5$$

$$x_1, x_2 \geq 0.$$

Solution:

The best and worst solutions for each objective function are getting as follows:

$-5.99 \leq Z_1 \leq -0.688$, $-31 \leq Z_2 \leq -0.272$, $-14.42 \leq Z_3 \leq -1.45$

This (MOLPLFPP) is identical to the QPP. The (MOLPLFP) issues can be composed as follows:

$$\text{Max. Z} = \{8x_1^2 - 5x_1x_2 - 60.162x_1 - 1.936x_2 - 21.04\}.$$

Solve this QPP with the same constraints we get:

$Max. Z = -21.04 \; at \; (0,0).$

4. Consider the MOLPLFP Problem

$$Max. Z_1 = (3x_1 + 2x_2) + \frac{(x_1 - x_2 + 2)}{(x_1 + x_2 + 2)}$$

$$Max. Z_2 = (x_1 + 2) + \frac{(2x_1 - 5x_2)}{(x_1 + 2x_2 + 3)}$$

$$Max. Z_3 = (x_2 + 7) + \frac{(5x_1 + 5x_2 + 1)}{(3x_1 + 3x_2 + 2)}$$

$$Max. Z_4 = (x_1 + x_2 + 5) + \frac{(x_1 + 1)}{(x_2 - 5)}$$

$$Max. Z_5 = (-x_1 + x_2 - 2) + \frac{(x_1 + x_2 + 2)}{(x_1 + x_2 + 1)}$$

Subject to: $x_1 + \; 2x_2 \leq 1$

$2x_1 + 3\,x_2 \leq 6$

$x_1 \qquad \leq 1$

$x_1, x_2 \geq 0.$

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Solution:

The best and worst solutions for each objective function are getting as follows:

$1 \leq Z_1 \leq 4$, $1.375 \leq Z_2 \leq 3.5$, $7.5 \leq Z_3 \leq 8.5$, $4.8 \leq Z_4 \leq 5.6$, $-1.5 \leq Z_5 \leq 0.167$

This (MOLPLFPP) is identical to QPP. the (MOLPLFP) issues can be composed as follows:

$$\text{Maximum Z} = \{3x_1^2 + 11x_1x_2 + 7\,x_2^2 + 0.833x_1 - 12.267x_2 + 8.667\}.$$

Solve this QPP with the same constraints we get:

$Max.Z = 12.5\ at\ (1,0).$

5. Consider the MOLPLFP Problem

$$Max.Z_1 = (x_1 + x_2 + 1) + \frac{(2x_1 - x_2)}{(2x_1 + 2x_2 + 2)}$$

$$Max.Z_2 = (x_1 - 1) + \frac{(2x_2 + 1)}{(x_1 - 5)}$$

$$Max.Z_3 = (x_1 + 2x_2 - 5) + \frac{(x_1 - x_2 + 1)}{(x_2 - 6)}$$

$$Max.Z_4 = (2x_1 + 2x_2) + \frac{(x_1 + x_2)}{(x_1 + x_2 + 3)}$$

$$Max.Z_5 = (x_1 - 3) + \frac{(-x_1 + x_2)}{(4x_1 + 4x_2 - 1)}$$

$$Max.Z_6 = (3x_1 - 2x_2) + \frac{(2x_1 + 2x_2)}{(x_1 + 1)}$$

$$Max. Z_7 = (x_1 + x_2 - 1) + \frac{(x_1+2)}{(x_2-4)}$$

$$Max. Z_8 = (x_2 + 1) + \frac{(x_2-3)}{(x_1+2)}$$

$$Max. Z_9 = (x_2 - 2) + \frac{(x_1+x_2)}{(x_1+1)}$$

Subject to: $x_1 - x_2 \geq 1$

$x_1 + 3x_2 \leq 3$

$x_1, x_2 \geq 0.$

Solution: The best and worst solutions for each objective function are getting as follows:

$2.5 \leq Z_1 \leq 4.75$, $-0.25 \leq Z_2 \leq 1.5$, $-4.33 \leq Z_3 \leq -2.67$, $2.25 \leq Z_4 \leq 6.5$, $-2.33 \leq Z_5 \leq -0.27$

$4 \leq Z_6 \leq 10.5$, $-0.75 \leq Z_7 \leq 0.75$, $0 \leq Z_8 \leq 0.8$, $-1.5 \leq Z_9 \leq -0.7$

This (MOLPLFPP) is identical to QPP. The problem (MOLPLFP) can be composed as follows:

$$\text{Max. Z} = \{12x_1^2 + 14x_1x_2 + 7x_2^2 - 37.02x_1 - 30x_2 - 1.19\}.$$

Solve it we get

$$\text{Max. Z} = -4.25\ (3, 0).$$

A Comparison Between the Proposed Method and Others

The problem (MOLPLFPP) is previously solved by several methods. To infer the ideal explanation to solve (MOLPLFPP), used value function and chebyshev goal programming approaches, where he found the optimum solution by each Z_i individually [21]. However (MOLPLFPP) illuminated with fuzzy objective programming by Taylor series approximation, where Euclidean distance function is utilized for getting compromise ideal solution individually [22]. In addition, the strategy is proposed for understanding (MOLPLFPP), for non-differentiable term happens in for non-differentiable term happens in constraints. And the solution is gotten by reducing (MOLPLFPP) to a multi-objective fractional programming problem (MOFPP) by using reasonable substitutions and also using programming theorems, at that point the solution of the original (MOLPLFPP) can be gotten through the solution of reduced MOFPP [16].

In our method, we first transformed MOL plus LFPP to QPP based on a hypothesis already considered by Dinkelbach [20], and after that solved QPP by, the results were found by using Lingo problems.

Conclusion

In this paper, we displayed an unused arrangement to the (MOLPLFPP). The solution is based on a theory studied previously in [20], with the assistance of this proposed theory, all (MOLPLFP) problem changed to a (SO) function. Furthermore (MOLPLFPP) transformed into QPP.

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