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Jordan Higher Triple Left Resp. Right Centralizers of Prime Γ-Rings

Afrah Mohammed Ibraheem[*] and Salah Mehdi Salih

Department of Mathematics – College of Education – Al-Mustansiryah University – Iraq

[*]afrah.wisam@uomustansiriyah.edu.iq

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Abstract

Through this paper we define the higher triple left resp. right centralizers of a Γ-ring Ǵ, and study some properties of Jordan higher triple left resp. right centralizers of Ǵ, addition to we prove that: every Jordan higher triple left resp- right centralizer of a Γ-ring Ǵ is higher triple left resp. right centralizer of Ǵ when Ǵ is a 2-torsion free prime gamma ring.

Keywords: Prime Γ-ring, Higher left centralizer, Higher triple left centralizer, Jordan higher triple left centralizer.

تمركزات جوردان الثلاثية العليا اليسرى واليمنى على الحلقات-Γ الاولية

أفراح محمد ابراهيم و صلاح مهدي صالح

قسم الرياضيات - كلية التربية - الجامعة المستنصرية

الخلاصة

خلال هذا البحث قمنا بتعريف التمركزات الثلاثية العليا اليسرى واليمنى للحلقات-Γ ، كما درسنا خواص تمركزات جوردان الثلاثية العليا اليسرى واليمنى للحلقات-Γ ، بالاضافة الى اننا قمنا ببرهنة ان كل تمركز جوردان ثلاثي عالي يساري او يميني يكون تمركز ثلاثي عالي يساري او يميني عندما تكون الحلقة-Γ طليقة الألتواء اولية من النمط -2 .

الكلمات المفتاحية: الحلقات-Γ الاولية، التمركزات العليا اليسرى، التمركزات الثلاثية العليا اليسرى، تمركزات جوردان الثلاثية العليا اليسرى.

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Introduction

In 1964 Nobusawa [1] presented the notion of gamma ring, and in 1966 Barnes [2] generalized the concept of gamma ring, J. Jing in [3] defined a derivation on Γ-ring.

Within this paper we present and researchch higher tiple left resp-right centralizer and Jordan higher triple left resp-right centralizer of a gamma ring Ǵ, and prove that every Jordan higher triple left resp. right centralizer of a 2-torsion free prime Γ-ring Ǵ is a higher triple left resp. right centralizer of Ǵ.

1. Higher Triple Left resp. Right Centralizer of Γ-Rings

In this section we present the definition of higher triple left resp. right centralizer of a Γ-ring Ǵ, and study some properties of Jordan higher triple left resp-right centralizer of Ǵ.

Definition 1.1: Let Ǵ be Γ-ring, and Ţ=(t_k) $_{k\epsilon N}$ be family of additive mapping of Ǵ, then Ţ is called a higher triple left resp. right centralizer of Ǵ if ∀ a,b,c ∈Ǵ, α ϵΓ and k∈N

$t_k(a\alpha b\beta c) = \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\,\beta t_{i-1}(c)$,

$(\,resp.\ t_k(a\alpha b\beta c) = \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\,\beta t_{i-1}(c)$.

Example 1.2: Let Ǵ bè Γ-ring, t = (t_n) $_{n\epsilon N}$ be a higher triple left centralizer of Ǵ, let S= $\{(a,b)|a, b \in$ Ǵ$\}$ and $\Gamma = \{(\alpha,\alpha)|\alpha \in \Gamma\}$ where the addition and multiplication defined on S by $(a_1,b_1) + (a_2,b_2) = (a_1 + a_2, b_1 + b_2)$ and

$(a_1,b_1)(\alpha,\alpha)(a_2,b_2) = (a_1\alpha a_2, b_1\alpha b_2)$, ∀ $a_1, a_2, b_1, b_2 \in$ Ǵ , $\alpha \in \Gamma$

Let T: Ś →Ś, T= $(Ţ_n)_{n\epsilon N}$ be a family of additive mappings on Ś defined by

T_n (a , b) = (t_n (a) ,$ŧ_n$(b)), ∀ (a, b) ∈ S, then Ţ is a higher triple left centralizer on Ś.

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Definition 1.3 [4]: Let Ɠ a gamma-ring, and Ţ= (t_n) $_{n\epsilon N}$ be a family of additive mapping of Ɠ, then Ţ is called a Jordan higher triple left resp. right centralizer of Ɠ if ∀ ɑ,b ∈Ɠ, α,β ∈Γ and k∈N

$t_k(a\alpha b\beta a) = \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta\, t_{i-1}(a).$

Lemma 1.4: Let T= (t_n)$_{n\epsilon N}$ be family of Jordan higher triple left resp. right centralizer of Γ-ring Ɠ, then ∀ ɑ,b,c ∈Ɠ, α,β ∈ Γ.

$$t_k(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c) + t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(a)$$

Proof: $t_n((a+c)\alpha b\beta(a+c)) = \sum_{i=1}^{k} t_i(a+c)\alpha t_{i-1}(b)\beta t_{i-1}(a+c)$

$$= \sum_{i=1}^{k} ((t_i(a) + t_i(c))\alpha\, t_{i-1}(b)\beta(t_{i-1}(a) + t_{i-1}(c))$$

$= \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) + t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c) + t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(a) + t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(c)$ (1)

On the other hand

$$t_k((a+c)\alpha b\beta(a+c)) = \sum_{i=1}^{k} t_i(a\alpha b\beta a + a\alpha b\beta c + c\alpha b\beta a + c\alpha b\beta c)$$

$= \sum_{i=1}^{k} t_i(a\alpha b\beta a) + t_i(c\alpha b\beta c) + t_k(a\alpha b\beta c + c\alpha b\beta)$

$= \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(a) + t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(c) + t_k(a\alpha b\beta c + c\alpha b\beta a)$ ... (2)

Comparing (1) and (2) we get

$$t_k(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c) + t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(a)$$

Definition 1.5: Let Ǵ be a Γ-ring, and T= $(t_k)_{k\epsilon N}$ a family of Jordan higher triple left resp. right centralizer of Ǵ. We define φ *as*

$$\varphi_k(a,b,c)_{\alpha,\beta} = t_k(a\alpha b\beta c) - \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c), \forall\, a,b,c \in Ǵ, \alpha,\beta \in \Gamma.$$

Remark 1.6:

We note that T= $(t_i)_{i\epsilon N}$ is higher triple left resp. right centralizer of Γ-ring Ǵ iff $\varphi_k(a,b,c)_{\alpha,\beta} = 0$, ∀ a,b.c ∈ Ǵ , α,β ϵ Γ and k∈N.

Lemma 1.7: Let T= $(t_k)_{k\epsilon N}$ be a family of Jordan higher triple left resp.right centralizer of a Γ-ring Ǵ, then ∀ a,b,c ∈ Ǵ, α,β ϵ Γ and k∈ N we have

i) $\varphi_k(a,b,c)_{\alpha,\beta} = -\varphi_k(c,b,a)_{\alpha,\beta}$

ii) $\varphi_k(a+x,b,c)_{\alpha,\beta} = \varphi_k(a,b,c)_{\alpha,\beta} + \varphi_k(x,b,c)_{\alpha,\beta}$

iii) $\varphi_k(a,b+y,c)_{\alpha,\beta} = \varphi_k(a,b,c)_{\alpha,\beta} + \varphi_k(a,y,c)_{\alpha,\beta}$

iv) $\varphi_k(a,b,c+z)_{\alpha,\beta} = \varphi_k(a,b,c)_{\alpha,\beta} + \varphi_k(a,b,z)_{\alpha,\beta}$

Proof:

We prove for example (ii)

$\varphi_k(a+x,b,c)_{\alpha,\beta}$= $t_k((a+x)\alpha b\beta c) - \sum_{i=1}^{k} t_i(a+x)\alpha t_{i-1}(b)\beta t_{i-1}(c)$

$= t_k(a\alpha b\beta c) + t_k(x\alpha b\beta c) - \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c) + t_i(x)\alpha t_{i-1}(b)\beta t_{i-1}(c)$

$=t_k(a\alpha b\beta c) - \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c) + t_k(x\alpha b\beta c) - \sum_{i=1}^{k} t_i(x)\alpha t_{i-1}(b)\beta t_{i-1}(c)$

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$= \varphi_k(a,b,c)_{\alpha,\beta} + \varphi_k(x,b,c)_{\alpha,\beta}$.

Proòf (i), (iii), and (iv) by the same way of prove (ii).

2. The Main Results

Lemma 2.1: Let Ǵ be a Γ-ring, and Ţ= $(t_k)_{k\epsilon N}$ be a family of Jordan higher triple left (resp. right) centralizer of Ǵ,

then ∀ a, b, c, ǵ ∈Ǵ, α, β, γ ϵΓ and k ∈ N

i) $\varphi_k(a,b,c)_{\alpha,\beta}\,\gamma t_{k-1}(ǵ)\gamma\,[t_{k-1}(a), t_{k-1}(b), t_{k-1}(c)]_{\beta,\alpha} = 0$

ii) $\varphi_k(a,b,c)_{\alpha,\beta}\alpha\, t_{k-1}(ǵ)\alpha\,[t_{k-1}(a), t_{k-1}(b),\, t_{k-1}(c)]_{\beta,\alpha} = 0$

iii) $\varphi_k(a,b,c)_{\alpha,\beta}\,\beta t_{k-1}(ǵ)\beta\,[t_{k-1}(a), t_{k-1}(b),\, t_{k-1}(c)]_{\beta,\alpha} = 0$

iv) $\varphi_k(a,b,c)_{\alpha,\alpha}\alpha\, t_{n-1}(ǵ)\alpha\,[t_{k-1}(a), t_{k-1}(b),\, t_{k-1}(c)]_{\alpha,\alpha} = 0$

Proof:

(i) We prove by induction on k ∈ N

If k = 1,

Let $w = a\alpha b\beta c\,\gamma ǵ\gamma\, c\beta b\alpha a + c\alpha b\beta a\gamma ǵ\gamma a\beta b\alpha c$

$t(w) = t(a\alpha b\beta c\gamma ǵ\gamma c\beta b\alpha a + c\alpha b\beta a\gamma ǵ\gamma a\beta b\alpha c)$

$= t(a\alpha b\beta c\,\gamma ǵ\gamma c\beta b\alpha a) + t(c\alpha b\beta a\gamma ǵ\gamma a\beta b\alpha c)$

$= t(a)\alpha b\beta c\,\gamma ǵ\gamma c\beta b\alpha a + t(c)\alpha b\beta a\gamma ǵ\gamma a\beta b\alpha c$ (1)

On the other hand

$t(w) = t(a\alpha b\beta c\gamma ǵ\gamma c\beta b\alpha a + c\alpha b\beta a\gamma ǵ\gamma a\beta b\alpha c)$

$= t(a\alpha b\beta c)\gamma ǵ\gamma c\beta b\alpha a + t(c\alpha b\beta a)\gamma ǵ\gamma a\beta b\alpha c$ (2)

Compare (1) and (2) we have

$0 = (t(a\alpha b\beta c) - t(a)\alpha b\beta c)\gamma ǵ\gamma c\beta b\alpha a + (t(c\alpha b\beta a) - t(c)\alpha b\beta a\,)\gamma ǵ\gamma a\beta b\alpha c$

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$0=\varphi(a,b,c)_{\alpha,\beta}\gamma ǵ\gamma c\beta b\alpha a+\varphi(c,b,a)_{\alpha,\beta}\gamma ǵ\gamma a\beta b\alpha c$

$0=\varphi(a,b,c)_{\alpha,\beta}\gamma ǵ\gamma c\beta b\alpha a-\varphi(a,b,c)_{\alpha,\beta}\gamma ǵ\gamma a\beta b\alpha c$

Then

$\varphi(a,b,c)_{\alpha,\beta}\ \gamma ǵ\gamma[a,b,c]_{\beta,\alpha}=0.$

Now, we can assume assumpition

$\varphi_s(a,b,c)_{\alpha,\beta}\ \gamma t_{s-1}(ǵ)\gamma\ [t_{s-1}(a),t_{s-1}(b),t_{s-1}(c)]_{\beta,\alpha}=0,\forall a,b,c,ǵ\in$ Ǵ, $\alpha,\beta,\gamma\epsilon\, \Gamma$ and s, k $\in N, s<k$.

Then according to definition 1.1 we get

$t_k(w)=t_k(a\alpha(b\beta c\gamma ǵ\gamma c\beta b)\alpha a+c\alpha(b\beta a\gamma ǵ\gamma a\beta b)\alpha c)$
$=\sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b\beta c\gamma ǵ\gamma c\beta b)\ \alpha t_{i-1}(a)\ +t_i(c)\alpha t_{i-1}(b\beta a\gamma ǵ\gamma a\beta b)\alpha t_{i-1}(c)$

$=\sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\ \beta t_{i-1}(c)\gamma t_{i-1}(ǵ)\gamma t_{i-1}(c)\beta t_{i-1}(b)\alpha t_{i-1}(a)\ +$
$t_i(c)\alpha t_{i-1}(b)\beta t_{i-1}(a)\gamma t_{i-1}(ǵ)\gamma t_{i-1}(a)\beta t_{i-1}(b)\alpha t_{i-1}(c)$

$=(\sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta\ t_{i-1}(c))\gamma\ t_{k-1}(ǵ)\gamma t_{k-1}(c)\beta t_{k-1}(b)\alpha t_{k-1}(a)\ +$
$\sum_{i=1}^{k-1} t_i(a)\alpha t_{i-1}(b)\beta\ t_{i-1}(c)\gamma t_{i-1}(ǵ)\gamma t_{i-1}(c)\beta t_{i-1}(b)\alpha t_{i-1}(a)$
$=(\sum_{i=1}^{k} t_i(c)\alpha t_{i-1}(b)\ \beta t_{i-1}(a))\ \gamma t_{n-1}(ǵ)\gamma t_{k-1}(a)\beta t_{k-1}(b)\alpha t_{k-1}(c)\ +$
$\sum_{i=1}^{k-1} t_i(c)\alpha t_{i-1}(b)\beta\ t_{i-1}(a)\gamma t_{i-1}(ǵ)\gamma t_{i-1}(a)\beta t_{i-1}(b)\alpha t_{i-1}(c)$ (3)

On the other hand, according to definition 1.1

$t_k(w)=t_k((a\alpha b\beta c)\gamma ǵ\gamma(c\beta b\alpha a)+(c\alpha b\beta a)\gamma ǵ\gamma(a\beta b\alpha c))$

$=\sum_{i=1}^{k} t_i(a\alpha b\beta c)\gamma t_{i-1}(ǵ)\ \gamma t_{i-1}(c\beta b\alpha a)\ +t_i(c\alpha b\beta a))\gamma t_{i-1}(ǵ)\gamma t_{i-1}(a\beta b\alpha c)$

$=t_k(a\alpha b\beta c)\gamma t_{k-1}(ǵ)\gamma\ \ t_{k-1}(c\beta b\alpha a)\ +\sum_{i=1}^{k-1} t_i(a\alpha b\beta c)\gamma t_{i-1}(ǵ)\gamma\ t_{i-1}(c\beta b\alpha a)\ +$
$t_k(c\alpha b\beta a))\gamma t_{k-1}(ǵ)\gamma t_{k-1}(a\beta b\alpha c)\ +\sum_{i=1}^{k-1} t_i(c\alpha b\beta a)\gamma t_{i-1}(ǵ)\gamma\ t_{i-1}(a\beta b\alpha c)$ (4)

Compare (3), (4), and by assumpition we get

$0=(t_k(a\alpha b\beta c)-\sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\,\beta t_{i-1}(c))\gamma t_{k-1}(ǵ)\gamma t_{k-1}(c)\beta t_{k-1}(b)\alpha t_{k-1}(a)$ +
$(t_k(c\alpha b\beta a)-\sum_{i=1}^{k} t_i(c)\alpha t_{i-1}(b)\beta\, t_{i-1}(a))\gamma t_{k-1}(ǵ)\gamma t_{k-1}(a)\beta t_{k-1}(b)\alpha t_{k-1}(c)$

Then it follows that

$0=\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma t_{k-1}(c)\beta t_{k-1}(b)\alpha t_{k-1}(a)+$
$\varphi_k(c,b,a)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma t_{k-1}(a)\beta t_{k-1}(b)\alpha t_{k-1}(c)$

$0=\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma t_{k-1}(c)\beta t_{k-1}(b)\alpha t_{k-1}(a)-$
$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma t_{k-1}(a)\beta t_{k-1}(b)\alpha t_{k-1}(c)$

Thus

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{n-1}(ǵ)\gamma[t_{k-1}(a),t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}=0$, ∀ a,b, $c, ǵ \in Ǵ$, $\alpha,\beta,\gamma \in \Gamma$ and k ∈ N.

(ii) We get it by substituting α for γ in (i).

(iii) We get it by substituting β for γ in (i).

(iv) We get it by substituting α for β and α for γ in (i).

Theorem 2.2:

Let Ǵ be a prime gamma ring, and Ţ= $(t_i)_{i\in N}$ a Jordan higher triple left resp. right centralizer of Ǵ, then ∀ a, b, c, ǵ, u, v, w ∈ Ǵ , α, β, γ ϵ Γ and k ∈ N ,

i) $\varphi_k(a,b,c)_{\alpha,\beta}\gamma\, t_{k-1}(ǵ)\gamma\, [t_{k-1}(u),t_{k-1}(v),t_{k-1}(w)]_{\beta,\alpha}=0$

ii) $\varphi_k(a,b,c)_{\alpha,\beta}\alpha\, t_{k-1}(ǵ)\alpha\, [t_{k-1}(u),t_{k-1}(v),t_{k-1}(w)]_{\beta,\alpha}=0$

iii) $\varphi_k(a,b,c)_{\alpha,\beta}\beta\, t_{k-1}(ǵ)\beta\, [t_{k-1}(u),t_{k-1}(v),t_{k-1}(w)]_{\beta,\alpha}=0$

iv) $\varphi_k(a,b,c)_{\alpha,\alpha}\alpha\, t_{k-1}(ǵ)\alpha\, [\,t_{k-1}(u),t_{k-1}(v),t_{k-1}(w)]_{\alpha,\alpha}=0$

Proof:

By replacing a+u for a in lemma 2.1 (i)

$\varphi_k(a+u,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma[\,t_{k-1}(a+u),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}=0$

Then

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(a),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}+$

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(u),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}+$

$\varphi_k(u,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(a),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}+$

$\varphi_k(u,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(u),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}=0$

Applying lemma 2.1(i) , we get

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(u),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}+$

$\varphi_k(u,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(a),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}=0,$

and

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(u),t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}=$

$-\varphi_k(u,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(a),t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}$

Therefore, we get

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(u),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}\,\gamma t_{k-1}(m)\gamma$

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(u),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}=0$

Hence

$-\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(u),t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}\gamma t_{k-1}(m)\gamma$

$\varphi_k(u,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(a),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}=0$

Since M is a prime, we have

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma\,[\,t_{k-1}(u),\ t_{k-1}(b),t_{k-1}(c)]_{\beta,\alpha}=0$ … (1)

Replacing b+v for b in lemma 2.1(i)

$\varphi_k(a,b+v,c)_{\alpha,\beta}\gamma t_{k-1}(\acute{g})\gamma[\,t_{k-1}(a),\ t_{k-1}(b+v),t_{k-1}(c)]_{\beta,\alpha}=0$

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Then

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(b),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}+$

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(v),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}+$

$\varphi_k(a,v,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(b),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}+$

$\varphi_k(a,v,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma[\mathrm{t}_{k-1}(a),\mathrm{t}_{k-1}(v),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}=0$

Applying lemma 2.1(i), we get

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(v),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}+$

$\varphi_k(a,v,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(b),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}=0$, and

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(v),\ \mathrm{t}_{k-1}(c)]_{\beta,\alpha}=$

$-\varphi_k(a,v,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(b),\ \mathrm{t}_{k-1}(c)]_{\beta,\alpha}$

Therefore, we get

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(v),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}\gamma t_{k-1}(ǵ)\gamma$

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(v),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}=0$

$-\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(v),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}$

$\gamma t_{n-1}(ǵ)\gamma\ \varphi_k(a,v,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(b),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}=0$

Since Ǵ is a prime, we have

$$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\mathrm{t}_{k-1}(a),\ \mathrm{t}_{k-1}(v),\mathrm{t}_{k-1}(c)]_{\beta,\alpha}=0 \qquad (2)$$

Replace c+w for c in lemma 2.1(i), and use the same way as in above we get

$$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma[t_{k-1}(a),t_{k-1}(v),t_{k-1}(w)]_{\beta,\alpha}=0 \qquad (3)$$

Now, replace a+u, b+v and c+w by a, b and c respectively we get

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma[t_{k-1}(a+u),t_{k-1}(b+v),t_{k-1}(c+w)]_{\beta,\alpha}=0$

Use the same way in above we get

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma[t_{k-1}(u),t_{k-1}(v),t_{k-1}(w)]_{\beta,\alpha}=0$

(ii) By replacing γ by α in (i).

(iii) By replacing γ by β in (i).

(iv) By replacing β by α and γ *by* α in (i).

Theorem 2.3: Let Ǵ be a 2-torsion free prime gamma ring. Then every Jordan higher triple left resp. right centralizer of Ǵ is higher triple left resp. right centralizer of Ǵ .

Proof:

Let $T=(t_i)_{i\in N}$ be a Jordan higher triple left resp. right centralizer of a prime Γ-ring Ǵ, then by theorem 2.2 (i), we have

$\varphi_k(a,b,c)_{\alpha,\beta}\gamma t_{k-1}(ǵ)\gamma\,[\,t_{k-1}(u),\ t_{k-1}(v),t_{k-1}(w)]_{\beta,\alpha}=0,\ \forall\ a,b,c,ǵ,u,v,w\in Ǵ,$ $\alpha,\beta,\gamma\in\Gamma$ and $k\in N$.

Since Ǵ is a prime then, either $\varphi_k(a,b,c)_{\alpha,\beta}=0$ or $[\,t_{k-1}(u),\ t_{k-1}(v),t_{k-1}(w)]_{\beta,\alpha}=0,\ \forall\ u,v,w\in Ǵ,\ \alpha,\beta\in\Gamma$ and $k\in N$.

If $[\,t_{k-1}(u),\ t_{k-1}(v),t_{k-1}(w)]_{\beta,\alpha}\neq 0$, then $\varphi_k(a,b,c)_{\alpha,\beta}=0$, and by remark 1.6 we get T is a higher triple centralizer of Ǵ.

If $\varphi_k(a,b,c)_{\alpha,\beta}\neq 0$ then $[\,t_{k-1}(u),\ t_{k-1}(v),t_{k-1}(w)]_{\beta,\alpha}=0$, then Ǵ is commutative, and by lemma 1.4 we have

$t_k(a\alpha b\beta c+a\alpha b\beta c)=\sum_{i=1}^{k}t_i(a)\alpha\,t_{i-1}(b)\beta\,t_{i-1}(c)+t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c)$

$t_k(2a\alpha b\beta c)=2\sum_{i=1}^{k}t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c)$.

Since Ǵ is a 2-torsion free, we get

$t_k(a\alpha b\beta c)=\sum_{i=1}^{k}t_i(a)\alpha t_{i-1}(b)\beta t_{i-1}(c),\ \forall\ a,b,c\in Ǵ,\alpha,\beta\in\Gamma$ and

$k \in N$. Therefore, T is a higher triple left resp. right centralizer of Ǵ.

Proposition 2.4: Every Jordan higher triple left resp-right centralizer of a prime

Γ-ring Ǵ is a higher left resp.right centralizer of Ǵ.

Proof:

Let $T = (t_i)_{i \in N}$ be a family of Jordan higher triple left (resp. right) centralizer of a prime Γ-ring Ǵ, then

$t_k((a\alpha b)\beta ǵ \beta(a\alpha b)) = \sum_{i=1}^{k} t_i(a\alpha b)\beta t_{i-1}(ǵ)\, \beta t_{i-1}(a\alpha b)$

$= \sum_{i=1}^{k} t_i(a\alpha b)\beta t_{i-1}(ǵ)\, \beta t_{i-1}(a)\alpha t_{i-1}(b)$ (1)

On the other hand

$t_k((a)\alpha b\beta ǵ \beta a\alpha b) = \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)\beta\, t_{i-1}(ǵ)\beta t_{i-1}(a)\alpha t_{i-1}(b)$ (2)

Comparing (1) and (2) we get

$0 = (t_k(a\alpha b) - \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b))\beta t_{k-1}(ǵ)\beta t_{k-1}(a)\alpha t_{k-1}(b)$,

$\forall$ a, b $\in$ Ǵ, $\dot\alpha, \beta \in \Gamma$ and $k \in N$.

Since Ǵ is a prime and a, b ≠ 0, we get

$t_k(a\alpha b) = \sum_{i=1}^{k} t_i(a)\alpha t_{i-1}(b)$. Hence T is a higher left centralizer of Ǵ.

Corollary 2.5: Every higher triple left resp. right centralizer on a prime Γ-ring Ǵ is a higher left (resp. right) centralizers on Ǵ.

Proof:

Let Ţ= $(t_k)_{k \in N}$ be a higher triple left centralizer of a gamma ring Ǵ, then T is a Jordan triple higher left (resp- right) centralizers on Ǵ. By prop. 2.4 we get T is a higher left centralizer on Ǵ.

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Conclusions

Within this paper, we study the relation among the higher triple left resp. right centralizer, Jordan higher triple left resp- right) centralizer and higher left resp. right centralizer of a prime gamma -ring Ǵ.

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