Modulating mathematical function to represent the magnetic field of the magnetic lens

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Abstract

In the present work we modulate a mathematical function to use it as a target function to represent the magnetic field distribution for the double pole-pieces magnetic lens. This function has, in fact, two optimization parameters. The only important parameter is the half half-width of the field for the proposed magnetic lens, which can affected objective properties, when the other one optimization parameter (lens length) is constant, which is the literature survey proved that unaffected on the lens properties. Results have clearly shown that the optimization parameter for current function has a considerable effect on the lens aberrations, and the reconstructed pole-pieces.

Keywords: Electron Optics, Magnetic lenses, Objective properties, Synthesis technique

تحوير دالة رياضية لتمثيل المجال المغناطيسي لعدسة مغناطيسية

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المستخلص

تم في هذا البحث تحوير دالة رياضية لاستخدامها كدالة هدف لتمثيل توزيع المجال المغناطيسي للعدسة المغناطيسية ثنائية القطب المتناظرة، إن الدالة المفترضة لها متغيران أمثليه، هما نصف نصف عرض المجال وطول العدسة. أثبتت الدراسات

السابقة أن نصف عرض النصف للمجال هو العامل الوحيد المؤثر على خواص العدسة الشيئية لقد تم دراسة تأثير هذا المتغير، على الخواص البؤرية الشيئية للعدسة والاقطاب المعاد بناءها.

الكلمات المفتاحية: بصريات الإلكترون، العدسات المغناطيسية، الخواص الشيئية، تقنية التوليف.

Introduction

The branch of physics that deals with the problem of charged particle motion in an electromagnetic field is called electron optics. This means, however, deflecting, forming and focusing flows of charge particles and producing images by means of electron and ions beams. Furthermore, electron optics comprises the investigation and exploring the physical and optical properties of electrons beams under the influence of electric and magnetic fields [1].

The synthesis technique is considered to be the best method for improving the work or performance of the magnetic lenses and it is a method that saves time in comparison with the analytical procedure which takes much more time.

In electron optics, the synthesis procedure of electron lenses optimization is based on the fact that, the first-order properties and aberrations of any imaging magnetic field can be calculated by using mathematical functions to approximate the magnetic field, several good mathematical functions exist for assigning the magnetic field distribution such as Gaussian model, Exponential model, Cosine model …etc. [2]. It is important to note that, the values of optical properties, aberrations and pole-piece shape depend on the mathematical distribution of field function, i.e. the optimum design of magnetic lenses depends on the optimization parameters of proposed formula to represent the optimum axial magnetic field distribution [3].

Many investigations have been carried out in the first of electron and ion optics to synthesis the parameter lens by using mathematical expressions to represent the magnetic scalar potential distribution along the lens axis, see for example[4]. The trajectory of the electron beam inside the lens has been represented by a mathematical formula, when the paraxial ray equation is solved for the assigned beam trajectory to obtain the magnetic field distribution of the lens, see for example [5].

Objective lenses play an important role in the charged particles devices since they which form the first image for the sample under the test. Unfortunately, these lenses are never perfect and exhibit different defects lead to the deterioration of the image quality. The most effective of these defects are the spherical and chromatic aberrations which cause the image to be blurred. The impossibility for correcting these two defects is led to what is called now optimization [6]. Optimization means the approaches or the procedures by which these defects may decrease to its minimal for a certain application.

The properties of the final projector lens in an electron microscope column are very important. Usually, the accelerated electron beam enters the final projector lens parallel to the optical axis.

Mathematical function

The new Model of mathematical function was suggested to be used as a target function to design the double pole-pieces magnetic lens , due to similarity of the distribution of magnetic field and the function curve. In addition the mathematic expression of this function includes the important parameter which is called (half-half width) of the magnetic field.

The new target function has been used to represent the magnetic field distribution of double pole-piece magnetic lens is given by the following relationship.

$$B_z(z) = \left(\frac{a}{\pi}\right)\left(\frac{a}{(2z - a^2)^2 + a^2}\right) \qquad (1)$$

Where $B_z(z)$ is the magnetic field distribution, the parameter a is the half half-width of the magnetic field distribution and z is the optical distance along the axis.

Also, it is important to investigate the scalar magnetic potential V according to the relation.

$$B_z = -\mu_o \text{grad}V \qquad (2)$$

Where μ_o is the magnetic space permeability and equal to $4\pi x10^{-7}$ $H.m^{-1}$. By using the analytical solution of Laplace's equation, the shape of the pole piece that would produce the desired field can be determined. For axially symmetric systems the electrostatic or magnetic scalar potential V(r,z) can be calculated by using the expansion [7]:

$$V(R_p, z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2} \left(\frac{R_p}{2}\right)^{2k} \frac{d^{2k}V(z)}{d_z^{2k}} \qquad (3)$$

Where R_p is the radial height of the pole-piece, V_P is the potential value at the pole-piece surface, which is equivalent to half of the lens excitation NI and V_z'' is the second derivative of the magnetic scalar potential with respect to the z-coordinate. By taking the first two terms of equation (3) under consideration, the equipotential surfaces are given by the formula [7]:

$$R_P(z) = 2\left[\frac{V(z) - V_P}{V''(z)}\right]^{\frac{1}{2}} \qquad (4)$$

Paraxial electron trajectories r(z) are computed numerically, using a fourth-order Runge-Kutta formula to solve the paraxial ray equation [8]:

$$r'' + \frac{\eta}{8V_r} B_z^2(z) r = 0 \qquad (5)$$

Where r'' is the second derivative of the electron beam trajectory, r is the electron beam trajectory, η is the charge -to- mass quotient of the electron, V_r is the relativistically-corrected accelerating voltage and the primes are signs of differentiation with respect to z.

The spherical and chromatic aberration coefficients C_s and C_c are computed numerically by using Simpson's rule to evaluate the aberration integrals [9]:

$$C_S = \frac{e}{128mV_r} \int_a^b \left[\left(\frac{3e}{mV_r}\right) + B_z^4 r_\alpha^4 + 8B_z'^2 r_\alpha^4 - 8B_z^2 r_\alpha^2 r_\alpha'\right] dz \qquad (6)$$

$$C_c = \frac{e}{8mV_r}\int_a^b B_z^2 r_\alpha^2 dz \qquad (7)$$

The primes denoted to the derivative with respect to z , limits of integration a , and b depends on the properties of the lens with if they objective or projector. In objective properties, the integration covers only the interval from object plane z_o to image plane z_i in spite of the magnetic field limits. While in projector properties, the integration covers the magnetic field limits from start point z_1 to end point z_2 of magnetic field. Where z_1 to z_2 represents the total range of z within which a finite lens field exists. And r_α is the solution of the paraxial-ray equation (5), with initial condition depending on the nature of the magnetic lens operation mode.

Results and discussion

The relationship between the magnetic field distribution $B_z(z)$ for different values of (a=1, 2, 3, 4, and 5 mm) at lens length (L= 20mm), shows in fig.(1). It can be seen that as the parameter a increases, so the lens excitation NI along the optical axis increases, and V_r increases too during the increasing of **a** as figure 2 indicates. It was found that the maximum value of magnetic field B_{max} for different values of **a** is constant at value (0.3183) Tesla, as shown in fig3.

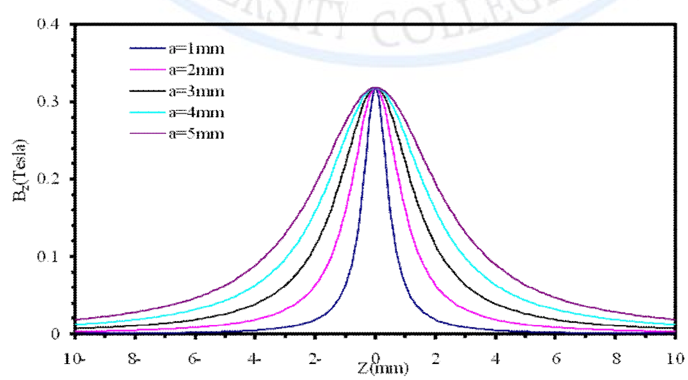

Figure 1: The magnetic field distribution B_z at different values of a.

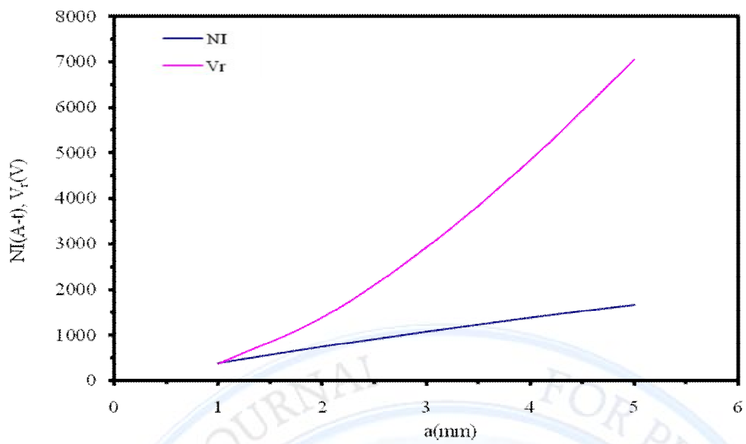

Figure 2: Magnetic field parameter NI and V_r as a function of a.

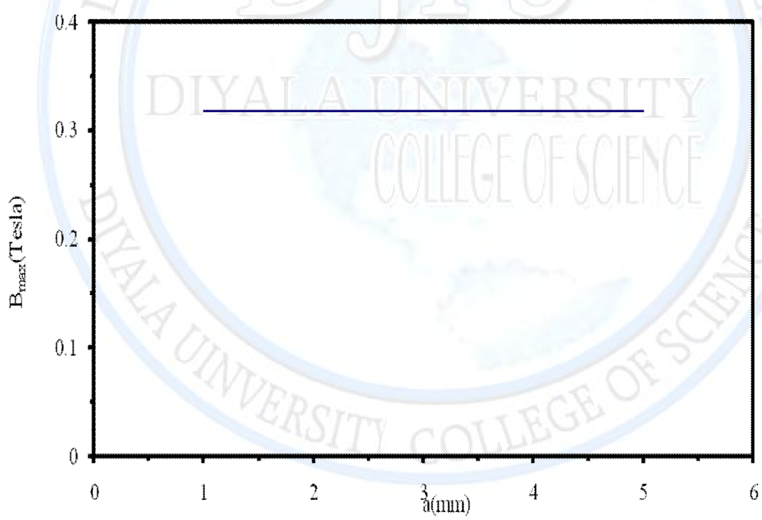

Figure 3: The maximum value of magnetic field B_{max} at different values of a.

Consequently the magnetic scalar potential at the terminal of the optical axis should increase too so as to satisfy the variation in NI and hence V(z) values at the optical axis terminals as shown in figure 4.

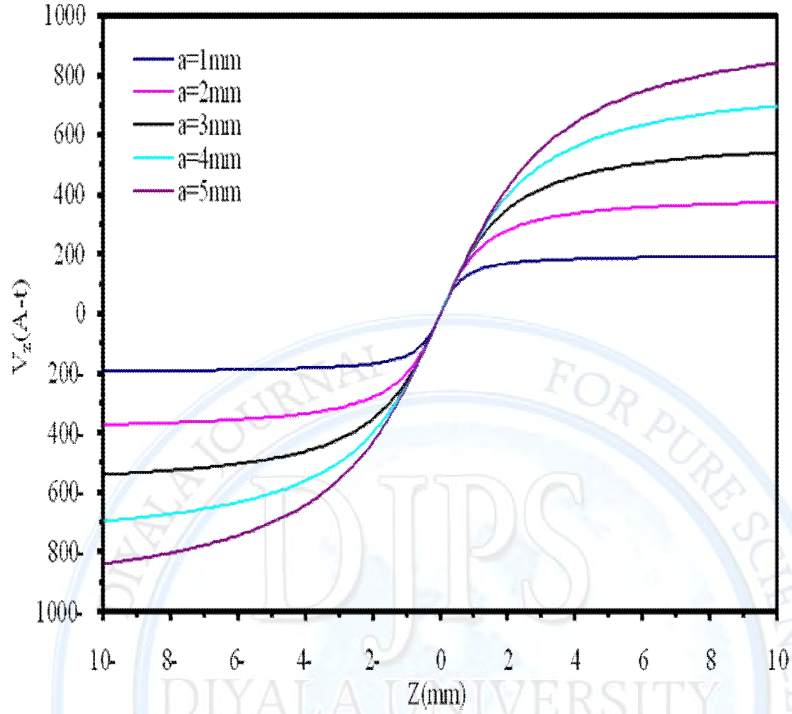

Figure 4: The magnetic scalar potential distribution (V_z) at different values of a.

The pole-piece profiles that can produce each B_z distribution, plotted in figure 1, are shown in figure 5. It can be seen that the consequences for increasing **a** lead to decreasing the pole-face curvature and hence increasing the air-gap width **s** and pole diameters D.

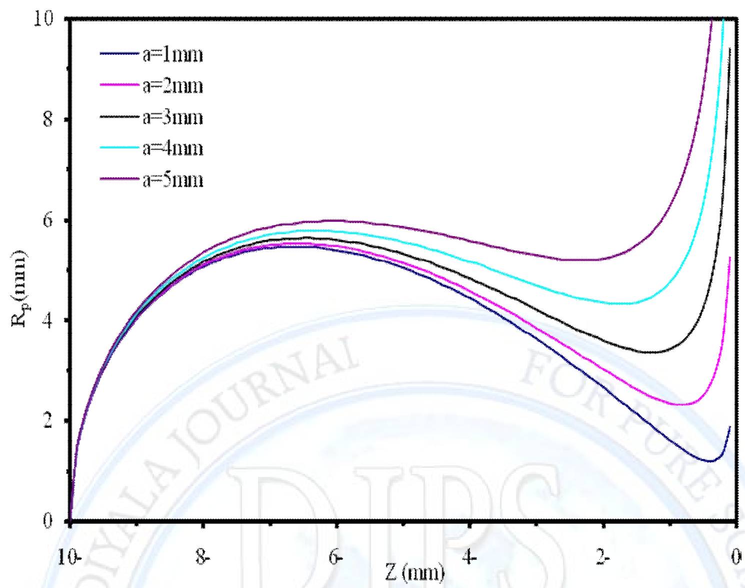

Figure 5: The upper-left quarter of the reconstructed pole-pieces for different values of a.

In the present work we investigate the objective properties at the excitation parameter $NI/V_r^{1/2}$=20.The variation of spherical and chromatic aberration coefficients together with objective focal length are plotted as a function of a in figure 6. Clearly it is seen that the objective focal length F_o increased with increasing half of the half-width a, as well as C_s and C_c increase, the values of the important objective focal properties of the objective lens F_o, C_s and C_c for various values of the parameter a are given in table 1.

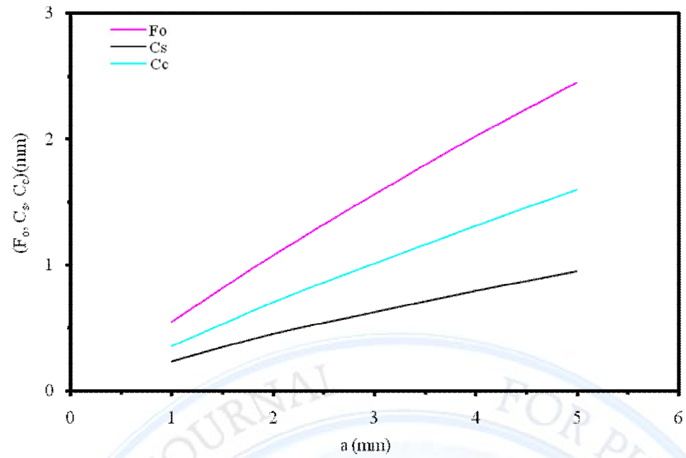

Figure 6: The spherical, chromatic aberration coefficients C_s, C_c, and the objective focal length F_o as a function of a.

Table 1: Some of the important parameters at different values of a.

a(mm)	NI(A-t)	V_r(V)	B_{max}(T)	F_o(mm)	C_s(mm)	C_c(mm)
1	385.2326	371.0105	0.3183	0.54618	0.23594	0.35688
2	745.2818	1388.613	0.3183	1.0735	0.4533	0.70186
3	1080.519	2918.804	0.3183	1.56059	0.62796	1.0118
4	1391.546	4840.997	0.3183	2.02194	0.79453	1.31229
5	1679.167	7049.004	0.3183	2.45056	0.95024	1.59825

Conclusions

1. In the present work we obtained a good results of the objective properties when small value of the half half width of the magnetic field distribution is used.

2. The behavior of the new target function gives constant maximum value of the magnetic field distributions for all values of the half half – width .

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